Advanced Quantum Physics, Exercises I

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1. Free propagator

The propagator $K(x, x', t)$ for the Hamiltonian H is defined through the solution of the Schrödinger equation

$$[i\hbar\partial_t - H(p, x)]\; K(x, x', t) = 0 \tag{1}$$

with the initial condition $K(x, x', 0) = \delta(x - x')$. Show that for an arbitrary initial condition $\psi_0(x)$ at $t = 0$ the solution of the Schrödinger solution is given by

$$\psi(x, t) \;=\; \int \; dx'\, K(x, x', t)\;\; \psi_0(x')\;. \tag{2}$$

Show, by using Fourier Transformation (plane waves expansion), that the propagator for free particles with $H = p^2/2m$ is given by

$$K(x, x', t) \;=\; \left(\frac{m}{2\pi\,\hbar\, i\, t}\right)^{1/2} \exp\left[\frac{im\,(x - x')^2}{2\hbar\, t}\right]\;. \tag{3}$$

Let now consider the following initial condition

$$\psi_0(x) = A\, e^{ik_0 x} e^{-\frac{(x-x_0)^2}{4\sigma}}\;. \tag{4}$$

(a) Calculate the norm A.

(b) Compute $\psi(x, t)$ using Eq.2

(c) Find the expectation values $\langle x \rangle$ and $\langle p \rangle$