Advanced Quantum Physics, Exercises V

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1. **Properties of angular momentum (Homework)**
   Compute the following commutators, using the fundamental commutation relations between position and momentum
   (a) $[L_x, L_y]$, what is the general form ?
   (b) $\left[\mathbf{L}^2, L_x\right]$,
   (c) $[L_y, \mathbf{p}^2]$,
   (d) $[L_z, x]$.
2. **Rotation of a di-amotic molecule (Oral)**
   To a good approximation we can describe a di-atomic molecule rotating in the rest frame of center of gravity at equal distance between the two rotating atoms (rigid Rotator). Due to the symmetry of the molecule we have two identical moments of inertia ($I_x = I_y \equiv I_\perp$), and a different third moment $I_z = I_\parallel$).
   (a) Give the Hamiltonian in terms of the angular momentum operators $\mathbf{L}^2$ und L_z.
   (b) Compute the eigenvaluess and eigenvectors.
3. **Expectation values of angular momentum (Oral)**
   We consider a system with the eigenstates $|l, m\rangle$ of $\mathbf{L}^2$ and L_z.
   (a) Calculate the expectation values $\langle L_x \rangle = \langle l, m | L_x | l, m \rangle$ and $\langle L_y \rangle$.
   (b) Calculate ΔL_x and ΔL_y.
   (Tip : Use $L_x = (1/\mathrm{i}\hbar)\,[L_y, L_z]$ and $L_y = (1/\mathrm{i}\hbar)\,[L_z, L_x]$. Note that L_z is hermitian.)