Advanced Quantum Physics, Exercises VI

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1. Pauli matrices properties (Oral)

The Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1}$$

Show the following relations:

a) $\sigma_i \sigma_j - \sigma_j \sigma_i = 2\mathrm{i}\epsilon_{ijk}\sigma_k$,

b) $\sigma_i \sigma_j = \mathrm{i}\epsilon_{ijk}\sigma_k$, $i \neq j$,

c) $\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij}$, and from that show $\sigma_i^2 = \mathbb{1}$ and $\sigma_i \sigma_j = -\sigma_j \sigma_i$,

d) $\sigma_i \sigma_j = \delta_{ij} + \mathrm{i}\epsilon_{ijk}\sigma_k$.

2. Spin resonance (Homework)

a) A spin in a external magnetic field is described by the Hamiltonian $H = \gamma \mathbf{B}(t)\mathbf{S}$. Show the following equation,

$$\frac{d}{dt}\boldsymbol{S}_H(t) = -\frac{\mathrm{i}}{\hbar}\left[\boldsymbol{S}_H, \hat{\mathrm{H}}_H\right] = \gamma \boldsymbol{S}_H(t) \wedge \boldsymbol{B}(t) \tag{2}$$

with $\gamma < 0$.

b) Solve the Heisenberg equation for an electron in a time-dependent magnetic field

$$\boldsymbol{B}(t) = \boldsymbol{B}_0 + B_1(\cos\omega t \boldsymbol{e}_x + \sin\omega t \boldsymbol{e}_y) \tag{3}$$

in which $\boldsymbol{B}_0 = (0, 0, B_0)$, $B_0 > 0$, $B_1 > 1$.

3. Hydrogen Atom (Homework)

The electron in a hydrogen atom has a spin $\mathbf{S}$ with $S = 1/2$, the orbital angular momentum $\mathbf{L}$, and nuclear spin $\mathbf{I}$ of the ionic core with $I = 1/2$

(a) The system is in the electronic ground state with orbital angular momentum $l = 0$. How large is the Ground state degeneracy? Which values of the total angular momentum $\mathbf{J} = \mathbf{S} + \mathbf{L} + \mathbf{I}$ are allowed and how large is the degeneracy of each angular momentum space?

(b) The system is electronically excited in a state with orbital angular momentum $l = 1$. Which values of the total angular momentum are now possible and how is the degeneracy?