# Advanced Quantum Physics, Exercises VII

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1. **Stark effect in harmonic oscillator (Oral)**
   The Hamiltonian of a 1-D harmonic oscillator in a homogeneous electric field $E$ is

   $$H = \frac{1}{2}(P^2 + Q^2) + eEQ. \tag{1}$$

   We consider the second part of the hamiltonian as a pertubation of the free oscillator $H_1 := Q$, with $\lambda := eE$.

   (a) Calculate the pertubed eigenvalues and eigenfunction up to the second order in $\lambda$.

   (b) Compare the perturbative result with the exact solution.

2. **Spin orbit and Zeeman effect (Homework)**
   We consider a system with angular momentum $L = 1$ and spin $S = 1/2$. Within $H_0$ all these spin states are degenerate. Now, we add the spin-orbit coupling $H_{so} = \gamma\mathbf{SL}$ as a perturbation.

   (a) Use the degenerate perturbation theory to determine the splitting of the degeneracy under the spin orbit coupling. (Hint : use the addition of angular momentum to find a appropriate basis. Note that $\mathbf{SL} = (\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2)/2$ with $\mathbf{J}$ the total angular momentum).

   (b) We now apply a magnetic field

   $$H = H_0 + H_{so} + H_m, \tag{2}$$

   where $H_m = -\vec{\mu}.\vec{B}$. Using perturbation theory solve this hamiltonian for two different cases :

   (i) weak magnetic field (Zeeman effect), consider here a pertubation of the spin orbit levels.

   (ii) strong magnetic field (Paschen-Back effect), where the spin orbit coupling can be ignored.

3. **Van-der-Waals interaction (Homework)**

   Consider two hydrogen atoms $(A, B)$ separated by a large distance $\boldsymbol{R}$, so that there is no overlap of electronic charge distribution, and have fixed nuclei (Born-Oppenheimer approximation). Let $H_\alpha$ be the hamiltonian of a single hydrogen atom. We can then write the Hamiltonian of the full system as

   $$H = H_A + H_B + H_{int}, \tag{3}$$

where $H_{int}$ describes the Coulomb interaction between the electrons and the positively charged nuclei, i.e.,

$$H_{int} = \frac{e^2}{R} + \frac{e^2}{|\boldsymbol{R} + \boldsymbol{r}_B - \boldsymbol{r}_A|} - \frac{e^2}{|\boldsymbol{R} + \boldsymbol{r}_B|} - \frac{e^2}{|\boldsymbol{R} - \boldsymbol{r}_A|}, \tag{4}$$

with $\boldsymbol{r}_i$ ($i \in A, B$) the position of an electron with respect to the nucleus $i$. For large $\boldsymbol{R} \gg a_0$ (Bohr-radius) one can develop $H_{int}$ in powers of $\boldsymbol{r}_i/\boldsymbol{R}$ and obtain the dipole-dipole interaction

$$H_{int} = \frac{1}{R^3} \left[ \boldsymbol{d}_A \boldsymbol{d}_B - \frac{3(\mathbf{d}_A \cdot \boldsymbol{R})(\boldsymbol{d}_B \cdot \boldsymbol{R})}{R^2} \right], \tag{5}$$

with the dipole operator $\boldsymbol{d}_i = e\boldsymbol{r}_i$. In the following we want to obtain the energy correction of the states $|g\rangle_A |g\rangle_B$ with the two atoms in the lowest energy state $|g\rangle_i$. We consider the distance $\mathbf{R}$ between the atoms as fixed.

a) Show that the first order in perturbation theory gives no contribution.

b) For the second order perturbation theory, we simplify the model, where we assume that the dipole operator couples only to excited state $|e\rangle_i$, i.e., $\langle e| \boldsymbol{d}_i |g\rangle_i = d/\sqrt{3}\mathbf{e}_z$ with $\mathbf{e}_z$ the unit vector along the z-axis. Show that the energy correction is always negative in the second-order and thus describes an attraction between the two atoms. Comment on the spatial dependence of the resulting interaction. Show that this interaction goes like $1/R^6$.

c) We now consider the degenerate excited p-states of the hydrogen atoms, i.e. , each atom has a 3-fold degenerate state $|2, 1, m\rangle$ with principal quantum number $n = 2$, angular momentum $l = 1$, and $m = 0, \pm 1$. Show the anistropic behavior of the Van der Waals interaction.