Boson coherent states

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17.04.2012

Motivation for Boson coherent states

- Eigenvectors of annihilation operator useful for second quantisation

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- Coherent states are important for a path integral formalism of many particle systems

Coherent states in 1D harmonic oscillator

- $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 = \hbar\omega(\overbrace{b^\dagger b}^{n} + \frac{1}{2})$, eigenstates $|n\rangle$
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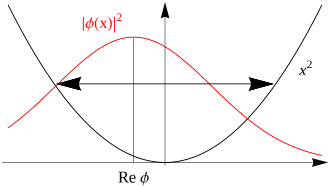

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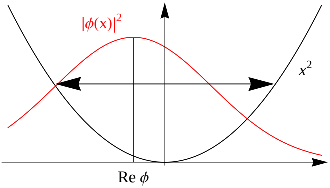
- $|\phi(x)|^2 = |\langle x|\phi\rangle|^2 \sim e^{-(x-\sqrt{2}\Re\phi)^2}$
- Minimal uncertainty $\sqrt{\mathsf{Var}(x)\mathsf{Var}(p)_\phi} = \frac{\hbar}{2}$



Repetition: Hilbert space and basis for N Bosons

- $\mathcal{H}^N = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \cdots \otimes \mathcal{H}^{(N)}$ with basis $|u^{(1)}_{i_1} \ldots u^{(N)}_{i_N}\rangle$

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- $|n_1 \dots n_j \dots\rangle \equiv \sqrt{\frac{N!}{\prod_j n_j!}} S |u_1^{n_1} \dots u_j^{n_j} \dots\rangle$

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- $\mathcal{F}$ is not restricted to N particles

Second quantisation for Bosons

- Definitions

$$\hat{n}_i|n_1 \dots n_i \dots\rangle = n_i|n_1 \dots n_i \dots\rangle$$

$$b_i|n_1 \dots n_i \dots\rangle = \sqrt{n_i}|n_1 \dots n_i - 1 \dots\rangle$$

$$b_i^\dagger|n_1 \dots n_i \dots\rangle = \sqrt{n_i + 1}|n_1 \dots n_i + 1 \dots\rangle$$

with

$$\hat{n}_i = b_i^\dagger b_i,\ [b_i, b_j^\dagger] = \delta_{ij},\ [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0,\ i = 1 \dots \infty$$

Creation operators $b_i^\dagger$

- $|n_1 \dots n_i \dots\rangle = \frac{(b_1^\dagger)^{n_1}}{\sqrt{n_1!}} \dots \frac{(b_i^\dagger)^{n_i}}{\sqrt{n_i!}} \dots |0\rangle$
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- Eigenvectors ?

$$b_i^\dagger|\Psi_s\rangle = \sum_{n_1 \dots n_j \dots} c_{n_1 \dots n_j \dots} b_i^\dagger |n_1 \dots n_j \dots\rangle$$

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- Fock space: vectors without hightest n_i possible

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- $\Rightarrow \phi_i c_{n_1 \dots n_i - 1 \dots} = \sqrt{n_i} c_{n_1 \dots n_i \dots}, \ \forall b_i$
- $\Rightarrow c_{n_1 \dots n_i \dots} = \phi_i^{n_i} \frac{1}{\sqrt{n_i!}} c_{n_1 \dots 0_i \dots n_j \dots} = \prod_i \phi_i^{n_i} \frac{1}{\sqrt{n_i!}} c_{0 \dots 0 \dots}$

- Eigenvectors for all b_i

$$|\phi\rangle = \sum_{n_1 \ldots n_j \ldots} c_{0\ldots0\ldots} \prod_l \frac{(\phi_l b_l^\dagger)^{n_l}}{n_l!} |0\rangle$$

$$= c_{0\ldots0\ldots} \exp\Big(\sum_l \phi_l b_l^\dagger\Big)|0\rangle$$

with $c_{0\ldots0\ldots} \equiv 1$ for simplicity

Boson coherent states

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- $b_i|\phi\rangle = \phi_i|\phi\rangle$, $\forall b_i$, $\{\phi_i\}_{i=1\dots\infty}$ sequence in $\mathbb{C}$
- For every single sequence $\{\phi_i\}_{i=1\dots\infty} \sim |\phi\rangle$ we can construct a coherent state!

Boson coherent states: properties

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- $\langle\phi|\theta\rangle = \langle 0|\exp\big(\sum_l \phi_l^* b_l\big)|\theta\rangle = \exp\big(\sum_l \phi_l^* \theta_l\big)\underbrace{\langle 0|\theta\rangle}_{1}$,
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- Demand $||\phi||^2 = \langle\phi|\phi\rangle = \exp\sum_i |\phi_i|^2 < \infty \Rightarrow$
  $\sum_i |\phi_i|^2 < \infty, \Leftrightarrow \{\phi_i\}_i \in \ell^2$
  $\Rightarrow \langle\phi|\theta\rangle < \infty$ (with Cauchy-Schwarz inequality)

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$$\int_{\mathbb{C}^\infty} \prod_{i=1}^{\infty} \left(\frac{1}{\pi} \mathrm{d}(\Re\phi_i) \mathrm{d}(\Im\phi_i) \right) \exp\left(-\sum_i |\phi_i|^2\right) |\phi\rangle\langle\phi| \stackrel{!}{=} \mathbb{1}_{\mathcal{F}}$$

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$$\overset{\phi=\rho e^{i\theta}}{=} \int \prod_{i=1}^{\infty} \left(\frac{\rho_i}{\pi} \mathrm{d}\rho_i \mathrm{d}\theta_i \exp\left(-\rho_i^2\right) \right) \left(\prod_k \exp\left(\phi_k b_k^\dagger\right) \right) |0\rangle\langle 0| \left(\prod_l \exp\left(\phi_l^* b_l\right) \right)$$

Overcompleteness: calculate i-th integral

$$\int_0^{2\pi}\int_0^{\infty}\frac{\rho_i}{\pi}\mathrm{d}\rho_i\mathrm{d}\theta_i e^{-\rho_i^2}\ \overbrace{\exp\left(\rho_i e^{i\theta_i} b_i^\dagger\right)}\ |0\rangle\langle 0|\ \overbrace{\exp\left(\rho_i e^{-i\theta_i} b_i\right)}$$

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$$= \sum_{n_i}|0\dots n_i\dots 0\dots\rangle\langle 0\dots n_i\dots 0\dots| = \mathbb{1}_{\mathcal{H}_i}$$

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$$\int_{\mathbb{C}^\infty} \prod_{i=1}^{\infty} \left(\frac{1}{\pi} \mathrm{d}(\Re\phi_i)\mathrm{d}(\Im\phi_i) \right) \exp\left(-\sum_i |\phi_i|^2\right) |\phi\rangle\langle\phi|$$

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- Why overcomplete? Linearly dependent!

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&= \int_{\mathbb{C}^\infty} \prod_{i=1}^{\infty} \left(\frac{1}{\pi} \mathrm{d}(\Re\phi_i) \mathrm{d}(\Im\phi_i) \exp\left(-|\phi_i|^2\right) \right) \langle\phi|A|\phi\rangle
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- $\langle\phi|f\rangle = f(\phi)$ the $\phi-$component of (not coherent) state f
  $\langle\phi|b_i^\dagger|f\rangle = \phi_i^* f(\phi)$, $\langle\phi|b_i|f\rangle = \partial_{\phi_i^*} f(\phi)$

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- Schrödinger equation

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$$H(\{b_i^\dagger, b_i\})|\Psi\rangle = E|\Psi\rangle \to H(\{\phi_i^*, \partial_{\phi_i^*}\})\Psi(\phi) = E\Psi(\phi)$$

$$\langle\phi|H|\Psi\rangle = \langle\phi| \left(\sum_{i,j} b_i^\dagger \langle i|T|j\rangle b_j + \frac{1}{2}\sum_{i,j,k,l} b_i^\dagger b_j^\dagger \langle ij|V|kl\rangle b_k b_l\right)|\Psi\rangle$$

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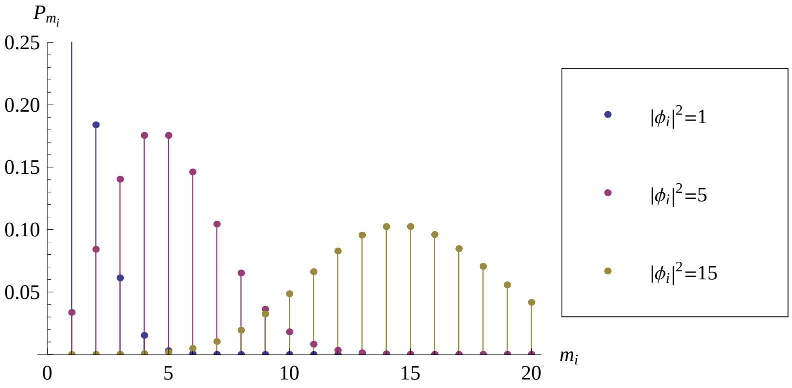
- Relative width $\frac{\sigma}{\langle N \rangle} = \frac{1}{\sqrt{\langle N \rangle}} \xrightarrow{\langle N \rangle \to \infty} 0$

Poisson distribution for fixed i



$$\langle N \rangle = \sum_k |\phi_k|^2$$

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