

Spontaneous symmetry breaking and field-theory II

Alejandro Muramatsu
Institut für Theoretische Physik III
Universität Stuttgart

Sommersemester 2014

Contents

1	Field-theory, vertex functions, and spontaneous symmetry breaking	5
1.1	Field-theory and statistical physics	5
1.1.1	The Ising model	5
1.1.2	Gaussian transformation	7
1.1.3	Continuum limit	8
1.1.4	Summary	12
1.2	Quantum field-theory	13
1.2.1	Quantum mechanics	13
1.2.2	Quantum field-theory in d-dimensions	16
1.3	Perturbative expansions, Feynman diagrams, and the free energy . . .	17
1.3.1	Non-interacting case	17
1.3.2	Interacting case - Feynman diagrams	20
1.3.3	Feynman rules	23
1.3.4	Connected propagators and the free energy	25
1.4	Vertex functions	28
1.4.1	Example: ϕ^4 theory	32
2	Continuous symmetries and Goldstone's theorem	37
2.1	Spin-systems with $O(n)$ -symmetry	37
2.1.1	Long-wavelength action for the $O(n)$ -Heisenberg model	38
2.2	Landau-theory and vertex function	40
2.3	Goldstone modes	41
2.4	Goldstone's theorem	43
3	The Mermin-Wagner theorem	45
4	Loop expansion and the renormalization group	51
4.1	Loop expansion and fluctuations around the classical limit	51
4.1.1	Zero loop: Landau-Ginzburg mean-field theory	52
4.1.2	One loop: Ginzburg's criterion	54
4.2	Dimensional analysis	56
4.2.1	Canonical dimensions	57

4.2.2	Power counting, critical dimension, and renormalizability	59
4.2.3	Relevant and irrelevant operators	64
4.3	Renormalization	65
4.3.1	Mass renormalization	65
4.3.2	Coupling constant renormalization	68
4.3.3	Wavefunction renormalization	72
4.3.4	Renormalization of a massive theory	75
4.3.5	Renormalization of a massless theory	75
4.3.6	Renormalization at $d < d_c$	77
4.4	The renormalization group	77
4.4.1	Renormalization of a massless theory	78
4.4.2	Callan-Symanzik equations for the renormalized theory	79
4.4.3	Fixed point, scaling and anomalous dimension	80
5	ϵ-expansion and the non-linear σ-model	85
5.1	Perturbation expansion for the non-linear σ -model	87
5.2	Dimensional regularization	89
5.2.1	Perturbative expansion with dimensional regularization	91
5.3	Renormalization group theory of the non-linear σ -model	94