

# Lattice gauge theory

Alejandro Muramatsu

Institut für Theoretische Physik III

Universität Stuttgart

Summer term 2014



# Contents

<b>1</b>	<b>Generalities on phase transitions</b>	<b>5</b>
1.1	Generalities on spontaneous symmetry breaking . . . . .	5
1.2	A phase transition without spontaneous symmetry breaking . . . . .	7
<b>2</b>	<b>The Ising model, duality, and transfer matrix</b>	<b>11</b>
2.1	Self-duality in the two-dimensional Ising model . . . . .	11
2.2	Transfer matrix: from two dimensional classical statistics to one dimensional quantum mechanics . . . . .	17
<b>3</b>	<b>Ising lattice gauge theory. Elitzur's theorem</b>	<b>23</b>
3.1	Ising model with a local $\mathbb{Z}_2$ symmetry . . . . .	23
3.2	Elitzur's theorem . . . . .	25
3.3	Gauge invariant correlation functions . . . . .	26
3.3.1	High temperature expansion . . . . .	27
3.3.2	Low temperature expansion . . . . .	28
3.4	Quantum Hamiltonian and duality transformation of the three-dimensional Ising gauge theory . . . . .	32
<b>4</b>	<b>Abelian lattice gauge theory</b>	<b>37</b>
4.1	U(1) lattice gauge theory . . . . .	37
4.2	Gauge-invariant correlation functions and phase diagrams . . . . .	39
4.2.1	Strong-coupling limit . . . . .	40
4.2.2	Weak-coupling limit . . . . .	41
4.2.3	Potential for static charges and confinement . . . . .	48
4.3	Two-dimensional Abelian lattice gauge theory . . . . .	49
4.4	The quantum Hamiltonian formulation and quark confinement . . . . .	52
<b>5</b>	<b>Non-Abelian lattice gauge theories</b>	<b>59</b>
5.1	General formulation of the SU(2) theory . . . . .	59
5.2	Discussion on non-Abelian theories . . . . .	62
5.3	A short overview on the renormalization group analysis of O(N) spin systems in two dimensions . . . . .	64
5.4	Coupling to matter fields in a lattice gauge theory . . . . .	70

