7. Problem: Disconnected pieces of diagrams diverge!

Example:

$$= \frac{1}{8}(-i\lambda)\int d^4z \, \underbrace{D_F(0)\,D_F(0)}_{\text{const}}$$

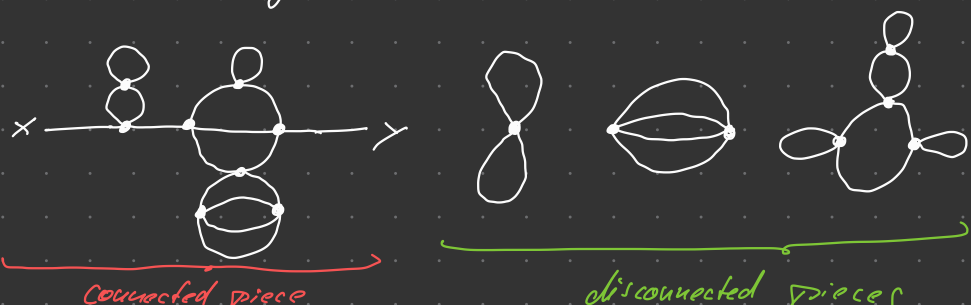
$$\propto (2T)\cdot(\text{Volume of Space } V)$$

Note:

$$\int d^4z = \lim_{T\to\infty(1-i\varepsilon)} \int_{-T}^{T} dz^0 \int d^3z$$

8. Exponentiation of disconnected diagrams:

a) Typical diagram:



b) Let

$$\mathcal{V} = \{V_1, V_2, \dots\} \equiv \left\{ \begin{array}{l} \text{Set of all disconnected} \\ \text{Feynman diagrams} \\ \text{without external points} \end{array} \right\}$$

$$\mathcal{F}^{xy} \equiv \left\{ \begin{array}{l} \text{Set of all connected} \\ \text{Feynman diagrams} \\ \text{with external points } x \text{ and } y \end{array} \right\}$$

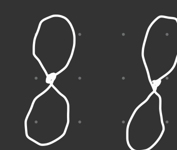
$\to$ Feynman diagram $F = \{ \underbrace{F^{xy}}_{\text{connected part}}, \underbrace{V_1, \dots, V_1}_{\text{multiplicity } n_1}, \underbrace{V_2, \dots, V_2}_{n_2}, V_3, \dots \}$

c) Amplitude of F:

$$F = F^{xy} \cdot \prod_i \underbrace{\frac{1}{n_i!}}_{S_i} (V_i)^{n_i}$$

S_i: Symmetry factor for exchanging the n_i copies of V_i.

Note:



$\to$ Sym. factor 2

V_1, $n_1 = 2$

d) Then

$$\langle 0|\mathcal{T}\{\phi(x)\phi(y)\, e^{-i\int dt\, H_I(t)}\}|0\rangle = \sum_{F\in\mathcal{F}^{xy}} \sum_{n_1,n_2,\dots} \left[F\cdot \prod_i \frac{1}{n_i!}(V_i)^{n_i} \right]$$

$$= \left[\sum_F{}' F\right] \times \left[\sum_{n_1,n_2\dots} \prod_i \frac{1}{n_i!}(V_i)^{n_i}\right]$$

$$= \left[\sum_F{}' F\right] \times \left[\prod_i \underbrace{\sum_{n_i} \frac{1}{n_i!}(V_i)^{n_i}}_{e^{V_i}}\right]$$

$$= \left[\sum_F{}' F\right] \times \exp\left[\sum_i V_i\right]$$

$$\rightarrow \quad \langle 0|\mathcal{T}\{\phi(x)\phi(y)\, e^{-i\int dt\, H_I(t)}\}|0\rangle = \Sigma(\mathcal{F}^{xy}) \times e^{\Sigma(\mathcal{V})}$$

with $\Sigma(X) \equiv \sum_{x\in X} x$

9. Denominator of (4.8):

$$\langle 0 | T\{ e^{-i\int dt\, H_I(t)} \} | 0 \rangle = e^{\Sigma(V)}$$

10. <u>Two-point correlator:</u>

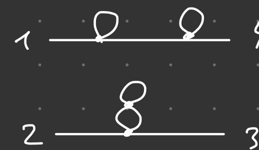
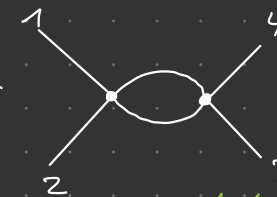
$$\boxed{\langle \Omega | T\{\phi(x)\phi(y)\} | \Omega \rangle = \Sigma(\mathcal{F}^{xy}) = \left\{ \begin{array}{l} \text{Sum of all connected diagrams} \\ \text{with two external points} \end{array} \right\}}$$

11. Generalization to <u>n-point correlators:</u>

$$\boxed{\langle \Omega | T\{\phi(x_1) \ldots \phi(x_n)\} | \Omega \rangle = \Sigma(\mathcal{F}^{x_1 \ldots x_n}) = \left\{ \begin{array}{l} \text{Sum of all connected diagrams} \\ \text{with } n \text{ external points} \end{array} \right\}}$$

Note 4.1

- <u>Connected diagrams</u> are connected to external points and not necessarily connected graphs:

$\langle \Omega | T \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle = \dots +$  $+ \dots +$ 

connected diagram
(but disconnected graph)

connected diagram
(and connected graph)

- Disconnected diagrams = "Vacuum bubbles"

- <u>Interpretation of vacuum bubbles:</u>

With (4.7) and (4.6):

$$\lim_{T \to \infty(1-i\varepsilon)} \langle 0 | T \left\{ \phi_I(x) \phi_I(y) \exp\left[-i \int_{-T}^{T} dt\, H_I(t) \right] \right\} | 0 \rangle = \overbrace{\langle \Omega | T \phi(x) \phi(y) | \Omega \rangle}^{\Sigma(\mathcal{F}^{xy})} \; e^{\Sigma(V)}$$

$$\times \lim_{T \to \infty(1-i\varepsilon)} \left\{ |\langle 0 | \Omega \rangle|^2 \, e^{-iE_0(2T)} \right\}$$

With $V_i = \tilde{V}_i \cdot (2T \cdot V)$:

$$\frac{E_0}{V} = i \sum_i \tilde{V}_i \qquad \text{(independent of } T\text{)}$$

V: Volume of space

→ total vacuum energy $E_0 \propto V$ (good!)

→ Vacuum bubbles determine the <u>vacuum energy density</u>