6. Example ($\phi_i \equiv \phi(x_i)$):

$$T\{\phi_1\phi_2\phi_3\phi_4\} = :\phi_1\phi_2\phi_3\phi_4 + \wick{\c\phi_1 \c\phi_2}\phi_3\phi_4 + \wick{\c\phi_1 \phi_2 \c\phi_3}\phi_4 + \wick{\c\phi_1 \phi_2 \phi_3 \c\phi_4}$$

$$+ \phi_1\wick{\c\phi_2 \c\phi_3}\phi_4 + \phi_1\wick{\c\phi_2 \phi_3 \c\phi_4} + \phi_1\phi_2\wick{\c\phi_3 \c\phi_4}$$

$$+ \wick{\c1\phi_1 \c1\phi_2 \c2\phi_3 \c2\phi_4} + \wick{\c1\phi_1 \c2\phi_2 \c1\phi_3 \c2\phi_4} + \wick{\c1\phi_1 \c2\phi_2 \c2\phi_3 \c1\phi_4} :$$

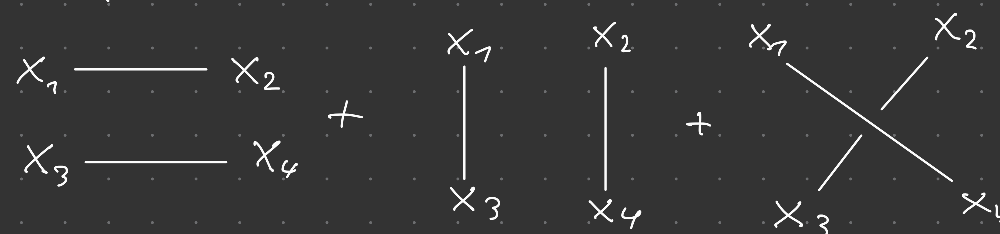
Therefore:

$$\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle = \langle 0|\, \wick{\c1\phi_1 \c1\phi_2 \c2\phi_3 \c2\phi_4} + \wick{\c1\phi_1 \c2\phi_2 \c1\phi_3 \c2\phi_4} + \wick{\c1\phi_1 \c2\phi_2 \c2\phi_3 \c1\phi_4}\, |0\rangle$$

$$= D_F(x_1-x_2)\,D_F(x_3-x_4) + D_F(x_1-x_3)\,D_F(x_2-x_4)$$

$$+ D_F(x_1-x_4)\,D_F(x_2-x_3)$$

=



4.4. Feynman Diagrams

(⇒ Problemset 6)

1. $\propto$ Numerator of (4.8)

$$\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle \propto \langle 0|T\left\{\phi(x)\phi(y)+\phi(x)\phi(y)\left[-i\int dt\, H_I(t)\right]+\mathcal{O}(\lambda^2)\right\}|0\rangle$$

2. λ^0-term: $\quad \langle 0|T\phi(x)\phi(y)|0\rangle = D_F(x-y) = x \;\text{———}\; y$

3. λ^1-term: $\quad \langle 0|T\left\{\phi(x)\phi(y)\,\frac{-i\lambda}{4!}\underbrace{\int dt\int d^3x}_{\int d^4z}\,\phi(z)\phi(z)\phi(z)\phi(z)\right\}|0\rangle$

Wick's theorem

$$= 3\cdot\frac{-i\lambda}{4!}\,D_F(x-y)\int d^4z\, D_F(z-z)\,D_F(z-z)$$

$$+\,12\,\frac{-i\lambda}{4!}\int d^4z\, D_F(x-z)\,D_F(y-z)\,D_F(z-z)$$

=

→ Interpretation:

Feynman diagram $\left\{ \begin{array}{lcl} \text{edges} = \text{"propagators"} & \longleftrightarrow & D_F \\ \text{internal points} = \text{"vertices"} & \longleftrightarrow & -i\lambda \int d^4 z \\ \text{external points} = \text{spacetime points} & \longleftrightarrow & x, y \end{array} \right\}$ Analytic expression

---

Feynman diagram $\hat{=}$ process of particle creation & propagation & annihilation

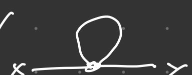
4. Prefactors:

- Feynman diagram = sum of all identical terms (= prefactor)
- $\measuredangle\, \mathcal{O}(\lambda^n)$ → factor $\frac{1}{n!}$ and n integrals / vertices

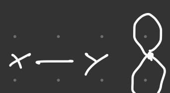
  → $n!$ possibilities to interchange vertices cancels $\frac{1}{n!}$

  → ignore the $\frac{1}{n!}$

- 4 contractions at each vertex
  - $\rightarrow$ $4!$ possibilities to interchange contractions
  - $\rightarrow$ $\frac{1}{4!}$ of interaction cancels $4!$
  - $\rightarrow$ associate $-i\lambda \int d^4z$ with each vertex
- Symmetries of diagrams reduce the number of different contractions
  - $\rightarrow$ divide expression by the symmetry factor S
- <u>Examples:</u>

$S\Big($  $\Big) = 2$ and $S = \Big($  $\Big) = 2 \cdot 2 \cdot 2 = 8$

Therefore:

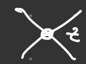
 $= \frac{1}{8} \cdot D_F(x-y)\,(-i\lambda) \int d^4z\, D_F(z-z)\, D_F(z-z)$

 $= \frac{1}{2} \cdot (-i\lambda) \int d^4z\; D_F(x-z)\, D_F(y-z)\, D_F(z-z)$

5. Therefore:

$$\langle 0|\,T\left\{\phi(x)\,\phi(y)\, e^{-i\int dt\, H_I(t)}\right\}|0\rangle = \sum \left\{\begin{array}{l}\text{Feynman diagrams with}\\ \text{two external points } x \text{ and } y\end{array}\right\}$$

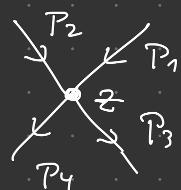
With position/real-space **Feynman rules** for the ϕ^4-theory:

1. For each edge, $\quad x \text{ — } y \; = D_F(x-y)$
2. For each vertex,  $\; = (-i\lambda)\int d^4 z$
3. For each external point, $\quad x \text{ —} \; = 1$
4. Divide by the symmetry factor, $\quad \frac{1}{S}\times \ldots$

6. Often calculations are simpler in momentum space:

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2-m^2+i\varepsilon} e^{-ip(x-y)}$$

Assign arbitrary orientation to edges and perform vertex integrals:

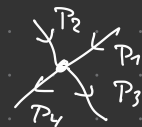


$$= (-i\lambda)\int d^4z \dots = (-i\lambda)(2\pi)^4\,\delta(p_1+p_2-p_3-p_4)$$

$\rightarrow$ Momentum conservation at vertices

→ Momentum-space Feynman rules

1. For each edge, $\xrightarrow{p} \;=\; \dfrac{i}{p^2 - m^2 + i\varepsilon}$

2. For each vertex, 

$= (-i\lambda)\,(2\pi)^4 \cdot \delta(p_1 + p_2 - p_3 - p_4)$

3. For each external point, $x \xleftarrow[p]{} \;=\; e^{-ipx}$

4. Integrate momenta, $\prod_i \int \dfrac{d^4 p_i}{(2\pi)^4} \cdots$

5. Divide by sym. factor, $\times \dfrac{1}{S} \cdots$