Solid State Theory, Exercises IV

Prof. Hans Peter Büchler SS 2012, 9th of May 2012

Exercise 1 - Low temperature properties of the Fermi gas

Based on the thermodynamic potential $\Omega(T, V, \mu)$ of the Fermi gas at low temperature, i.e., Eq. 4.247 in the lecture notes, derive the chemical potential μ in terms of E_F and T up to $O(T^2)$ and the specific heat c_V in terms of E_F and T up to $O(T)$.

From thermodynamics, we know the pressure P can be calculated from the thermodynamic potential Ω as $P = -\Omega/V$, derive the pressure of Fermi gas in terms of E_F and T up to $O(T^2)$, observe even at $T = 0$, there is residue pressure in the Fermi gas, where does this pressure come from ?

Exercise 2 - The static structure factor

The density of a system is defined as the (thermal or quantum) average $N/V = \langle \rho(\mathbf{r}) \rangle = \langle \sum_\alpha \delta(\mathbf{r} - \mathbf{r}_\alpha) \rangle$. Show that the structure factor

$$S(\mathbf{q}) \equiv \frac{1}{N} \langle \rho(\mathbf{q}) \rho(-\mathbf{q}) \rangle \tag{1}$$

and the pair distribution function in a homogeneous system

$$g(\mathbf{r}) \equiv \frac{\langle \rho(\mathbf{r}) [\rho(\mathbf{0}) - \delta(\mathbf{r})] \rangle}{\langle \rho(\mathbf{r}) \rangle \langle \rho(\mathbf{0}) \rangle} = \frac{V}{N^2} \langle \sum_{\alpha \neq \beta} \delta(\mathbf{r} - \mathbf{r}_\alpha + \mathbf{r}_\beta) \rangle \tag{2}$$

are related by

$$S(\mathbf{q}) - 1 = \frac{N}{V} \int d^3 r \; e^{i\mathbf{q} \cdot \mathbf{r}} \, g(\mathbf{r}). \tag{3}$$

For the case of a Fermi gas at zero temperature, write expressions for $S(\mathbf{q})$ and $g(\mathbf{r})$ in terms of the creation and annihilation operators, and sketch these functions.

Solutions due on : 18th of May, 2012