Solid State Theory, Exercises V

Prof. Hans Peter Büchler SS 2012, 16th of May 2012

Dynamical Structure Factor

(a) Prove the following useful mathematical relation

$$\lim_{\epsilon \to 0} \frac{1}{\omega \pm i\epsilon} = \mathcal{P}\frac{1}{\omega} \mp i\pi\delta(\omega), \tag{1}$$

where $\mathcal{P}$ denotes the principal value.

(b) The dynamic structure factor $S(\mathbf{q}, \omega)$ is defined as

$$S(\mathbf{q}, \omega) = \sum_n |\langle n| \rho_{-\mathbf{q}} |0\rangle|^2 \delta(\omega - \omega_0). \tag{2}$$

Using the above equation, demonstrate the following important relation between the dynamic structure factor and the imaginary part of the response function $\chi(\mathbf{q}, \omega)$,

$$Im\chi(\mathbf{q}, \omega) = -\pi[S(\mathbf{q}, \omega) - S(\mathbf{q}, -\omega)] \tag{3}$$

Dynamical Structure Factor of free electrons

(a) Determine the dynamic structure factor for non-interacting electrons and plot the results.

(b) Check the validity of the f-sum rule for the above results.

(c) Derive the compressibility of a Fermi gas from the compressibility sum rule.

(d) Demonstrate that this compressibility agrees with the compressibility derived from the thermodynamic properties at $T = 0$ via the relation

$$\kappa = \frac{1}{nN}\frac{\partial N}{\partial \mu}. \tag{4}$$

Solutions due on : 25th of May, 2012