Solid State Theory, Exercises VI

Prof. Hans Peter Büchler SS 2012, 23rd of May 2012

Landau-Fermi liquid theory

1. **Quasiparticle energy at finite temperature**
   The distribution function of a Fermi-liquid in thermodynamic equilibrium is given by
$$n_{\mathbf{p}} = \frac{1}{1+e^{(\tilde{\epsilon_{\mathbf{p}}}-\mu)/T}}, \tag{1}$$
   in which $\tilde{\epsilon_{\mathbf{p}}} = \epsilon_{\mathbf{p}} + \sum_{\mathbf{p}'} f_{\mathbf{p},\mathbf{p}'}\delta n_{\mathbf{p}'}$ is the quasiparticle energy and $\delta n_{\mathbf{p}} = n_{\mathbf{p}} - n^0_{\mathbf{p}}$, with $n^0_{\mathbf{p}} = \theta(|\mathbf{p}| - p_F)$.
   Estimate that $\sum_{\mathbf{p}'} f_{\mathbf{p},\mathbf{p}'}\delta n_{\mathbf{p}'}$ is proportional to $T^2 V g(\mu) f/m^* v_F^2$, with $g(\mu) = m^*\mathbf{p}_F^2/\pi^2$ and f the energy scale for $f_{\mathbf{p},\mathbf{p}'}$.
   Show that at low temperatures, we can use $\tilde{\epsilon_{\mathbf{p}}} = \epsilon_{\mathbf{p}}$ for Equation (1).
2. **Quasiparticle current**
   The transport equation for the distribution function, in the absence of external forces and collision, has the form :
$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \nabla_{\mathbf{r}} n_{\mathbf{p}} \cdot \nabla_{\mathbf{p}}\tilde{\epsilon_{\mathbf{p}}} - \nabla_{\mathbf{p}} n_{\mathbf{p}} \cdot \nabla_{\mathbf{r}}\tilde{\epsilon_{\mathbf{p}}} = 0. \tag{2}$$
   $\nabla_{\mathbf{p}}\tilde{\epsilon_{\mathbf{p}}}$ is the velocity of the quasiparticle, and $-\nabla_{\mathbf{r}}\tilde{\epsilon_{\mathbf{p}}}$ the force felt by the particle. Substituting $n_{\mathbf{p}}(\mathbf{p},t) = n^0_{\mathbf{p}} + \delta n_{\mathbf{p}}(\mathbf{r},t)$ and considering the terms up to first order in $\delta n_{\mathbf{p}}$ we get
$$\frac{\partial \delta n_{\mathbf{p}}}{\partial t} + \nabla_{\mathbf{r}}\delta n_{\mathbf{p}} \cdot \mathbf{v}_{\mathbf{p}} - \nabla_{\mathbf{p}} n^0_{\mathbf{p}} \cdot \sum_{\mathbf{p}'} f_{\mathbf{p},\mathbf{p}'}\nabla_{\mathbf{r}}\delta n_{\mathbf{p}'} = 0 \tag{3}$$
   a transport equation for the quasiparticle. If we interpret this as a continuity equation for the current and density of quasiparticle we find the quasiparticle current
$$\mathbf{j}_{\mathbf{p}} = \mathbf{v}_{\mathbf{p}} - \sum_{\mathbf{p}'} f_{\mathbf{p},\mathbf{p}'}\frac{\partial n^0_{\mathbf{p}'}}{\partial \epsilon_{\mathbf{p}'}}\mathbf{v}_{\mathbf{p}'}. \tag{4}$$
   The second term has the meaning of "drag" or "back flow" current, in fact because of the interaction the quasiparticle drags the medium along. The derivation of the quasiparticle current from the Galilean invariance, which applies only in translation-invariant systems, shows the origin of the "drag current". In a Galilean transformation, the momentum of the particle changes by $\mathbf{q}\,(ie\,\mathbf{p} \to \mathbf{p}+\mathbf{q})$ where $\mathbf{q}/m = \mathbf{v}$ is the velocity of the relative inertial system. The generated kinetic energy change of the system is therefore to the first order in $\mathbf{q}$,
$$\delta E = \langle \sum_i \mathbf{q} \cdot \frac{\mathbf{p}_i}{m} \rangle, \tag{5}$$
   from which the total current follows as $\mathbf{J} = \partial E/\partial \mathbf{q}$. We use this relation now on the state of an excited quasiparticle. Under Galilean-Transformation we alter both the quasiparticle and its momentum, as well as the occupation of the Fermi sea.
   Calculate the energy change of the system, and from it the quasiparticle current $\mathbf{j}_{\mathbf{p}}$.

3. **Effective mass**
In a translation-invariant system, the current of quasiparticle is $\mathbf{j_p} = \mathbf{p}/m$ in both interacting and non-interacting system. Using the above quasiparticle current (Eq 4), determine the effective mass $m^* = m(1 + F_1^s/3)$. The condition of stability for the system is $F_1^s > -3$.

4. **Spin-susceptibility**

(a) Derive the paramagnetic Spin-susceptibility at $T = 0$ for a free electron-gas

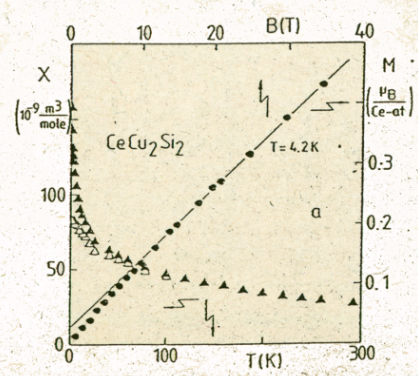
$$\chi^0 = g^0(\mu)\mu_B^2 = \frac{3}{2}\frac{n}{\epsilon_F^0}\mu_B^2. \tag{6}$$

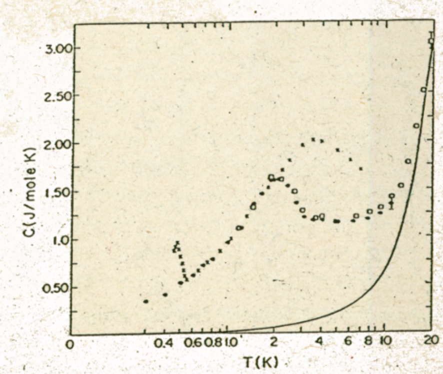
(b) Show that within Landau-Fermi liquid theory the susceptibility ($T = 0$) is given by

$$\chi = \frac{g(\mu)}{1 + F_0^a}\mu_B^2 = \frac{1 + F_1^s}{1 + F_0^a}\chi^0 \tag{7}$$

(c) From the experimental measurement of the specific heat and magnetic susceptibility of $CeCu_2Si_2$ one can obtain γ (through $C_v = \gamma T$) and $\chi(T = 0)$. What values do you get for γ and $\chi(T = 0)$, and what is the order of magnitude for m^* for this material ?

(d) The Wilson-Ratio, $R = \frac{T\chi(T=0)}{C_v}$ should be for a Fermi liquid around $R \sim 1$. R is of interest, since the m^* dependence has been eliminated from R. In the unit used in this data set we have $R = \frac{T}{C_v}\frac{\chi(T=0)}{\mu_0}\frac{\pi^2 k_B^2}{\mu_{eff}}$ ($\mu_0 = 4\pi \cdot 10^{-7} Hm^{-1}$). Due to the additional crystal fields one uses an effective $\mu_{eff} = 1.62\mu_B$. If (anti)-ferromagnetic fluctuations exist, they are found in F_0^a.
Determine the Wilson-Ratio for $CeCu_2Si_2$.





Solutions due on : 15th of June, 2012