Solid State Theory, Exercises VIII

Prof. Hans Peter Büchler SS 2012, 29th of June 2012

The RPA dielectric function

The general form of the screened susceptibility is given by

$$\chi_0(\mathbf{q},\omega) = 2\sum_{\mathbf{k}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}}}{\omega - \epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}} + i\eta} \tag{1}$$

(a) Show that the screened susceptibility is :

$$Re[\chi_0(\mathbf{q},\omega)] = -g\left\{\frac{1}{2} + \frac{k_F}{4q}\left[\frac{(\omega+q^2/2m)^2}{(qv_F)^2} - 1\right]\ln\left[\frac{\omega - qv_F + q^2/2m}{\omega + qv_F + q^2/2m}\right] - \frac{k_F}{4q}\left[\frac{(\omega-q^2/2m)^2}{(qv_F)^2} - 1\right]\ln\left[\frac{\omega - qv_F - q^2/2m}{\omega + qv_F - q^2/2m}\right]\right\},$$

and

$$Im[\chi_0(\mathbf{q},\omega)] = \begin{cases} -(1+g)\frac{\pi}{2}\frac{\omega}{qv_F} & \text{for } 0 < \omega < v_F q - q^2/2m \\ (1+g)\frac{\pi}{2}\frac{p_F}{q}\left[1 - \frac{(\omega - q^2/2m)^2}{(qv_F)^2)}\right] & \text{for } v_F q - q^2/2m < \omega < v_F q + q^2/2m \\ 0 & \text{for } v_F q + q^2/2m < \omega < \infty, \end{cases} \tag{2}$$

where g is the density of states.

(b) Using the above formula compute the dielectric function $\epsilon_{RPA}(\mathbf{q},\omega)$.

(c) Write down $Re[\epsilon_{RPA}]$ in the static limit $\omega \to 0$, and sketch it as a function of q. Expand at small q to recover the Thomas-Fermi screening form. What happens at $q = 2k_F$? Show that at this point the dielectric constant is continuous but non-differentiable. Explain why this a special value of q.

(d) Expand $Re[\epsilon_{RPA}(\mathbf{q},\omega)]$ at small q to order q^2 (this requires expansion of the logarithms to quite a high order in q/ω). You should recover the plasmon dispersion given in exercise sheet VII.

Solutions due on : 6th of July, 2012