Theoretische Physik III: Klassische Elektrodynamik, Exercise 6

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1. Magnetic moment of a rotating spherical shell (Written) [3pt]

A spherical shell of radius R and charge Q (homogeneously distributed on the surface) is rotating around its z-axis with angular velocity $\boldsymbol{\omega} = \omega \mathbf{e}_z$.

a) Calculate the current density $\mathbf{j}(\mathbf{r}) = \mathbf{v}(\mathbf{r})\rho(\mathbf{r})$.

b) Calculate the magnetic moment $\mathbf{m} = \frac{1}{2}\int d^3r(\mathbf{r} \times \mathbf{j}(\mathbf{r}))$ of the spherical shell.

c) Now let $\mathbf{r}_2$ be a vector such that $\mathbf{r}_2 \perp \boldsymbol{\omega}$ and $|\mathbf{r}_2| \gg R$. Calculate the lowest order contribution of the force exerted by the magnetic field

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3(\mathbf{r}\cdot\mathbf{m})\mathbf{r}}{|\mathbf{r}|^5} - \frac{\mathbf{m}}{|\mathbf{r}|^3}\right) = \frac{\mu_0 Q R^2}{12\pi}\left(\frac{3(\mathbf{r}\cdot\boldsymbol{\omega})\mathbf{r}}{|\mathbf{r}|^5} - \frac{\boldsymbol{\omega}}{|\mathbf{r}|^3}\right). \tag{1}$$

on another identical sphere placed at a point $\mathbf{r}_2$ and rotating with angular velocity $\boldsymbol{\omega}_2$ parallel to $\boldsymbol{\omega}$. Due to the large distance between the spheres you can approximate them as two point-like objects carrying some magnetic moment.

2. Parallel plate capacitor with a dielectric (Oral)

Consider a parallel plate capacitor with quadratic plates of edge length l and distance d between the plates. The capacitor is directly connected to a battery with voltage V.

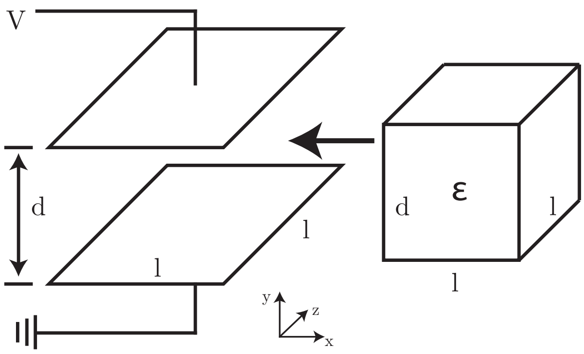

Figure 1: Parallel plate capacitor with dielectric media.

a) What is the total charge Q, respectively $-Q$ of both plates?

b) Now the dielectric media, length of the edges $d \times l \times l$ and isotropic dielectric constant $\varepsilon > 1$, is inserted into the capacitor, right in between the parallel plates. How does Q change, if the battery is still connected? What force acts on the dielectric media and in which direction?

c) This time we disconnect the plates from the battery before we insert the dielectric media. What happens now to the potential difference V and what force acts now?

Hint: Neglect boundary effects, i.e. consider a homogeneous electric field between the plates, which vanishes outside the plates.

3. Dielectric half space (Oral)

Consider two linear dielectric half spaces, which have the y-z-plane as common interface. For $x > 0$ there is the dielectric constant ε_1 and for $x < 0$ there is ε_2. Within the media 1 (ε_1) there is a point charge q. Calculate the **E** - and **D** -fields for the two cases $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_1 < \varepsilon_2$ by using the methods of image charges. In addition, sketch the **E** - and **D** -fields for both cases.

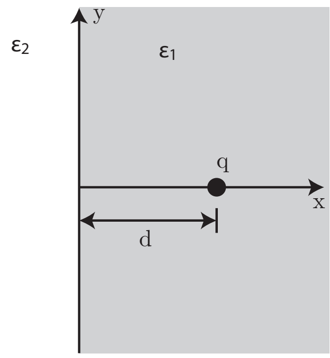

Figure 2: Half spaces separated by an interface, including a charge q.