# Theoretische Physik III: Klassische Elektrodynamik, Exercise 8

*Prof. Dr. Hans Peter Büchler, SS 2015, 10. June 2015*

1. **Wave in a medium (Oral)**

   Consider a plane wave in a non-conductive medium (in which the conductivity $\sigma$ disappears and $\epsilon$ and $\mu$ are constant).

   a) Derive the wave equation for the electromagnetic field directly from the Maxwell equations. Show that the plane wave

   $$\begin{pmatrix} \mathbf{E}(\mathbf{x},t) \\ \mathbf{B}(\mathbf{x},t) \end{pmatrix} = \begin{pmatrix} \mathbf{E}_0 \\ \mathbf{B}_0 \end{pmatrix} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{1}$$

   is a solution of these equations and determine the dispersion relation. At what speed does the wave propagate in the medium and at what speed in the vacuum?

   Next, we analyze the propagation of light in a metal.

   b) Derive the dielectric function $\epsilon(\omega)$ (also called dielectric permittivity) of a conductor in SI units:

   $$\epsilon(\omega) = \epsilon_0 + \frac{i\sigma(\omega)}{\omega}, \tag{2}$$

   where the conductivity has the form

   $$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}. \tag{3}$$

   c) How does the wave equation for the electromagnetic field look like? Show that the plane wave which penetrates the conductive material is attenuated (abgeschwächt). Calculate the penetration depth $\delta$ for a monochromatic wave in the limit of low frequencies. What conclusions can be drawn from this result?
   *Hint:* The penetration depth $\delta$ is defined as the distance at which the initial wave is attenuated by $e^{-1}$.

2. **Interplanetary sailing (Written) [3pt]**

   A planar electromagnetic wave, which spreads in the vacuum, reaches a perfectly conducting flat screen (later called “solar sail”) perpendicularly. The energy flux density (energy per unit area, per unit time) transported by the electromagnetic fields is given by the Poynting vector

   $$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}), \tag{4}$$

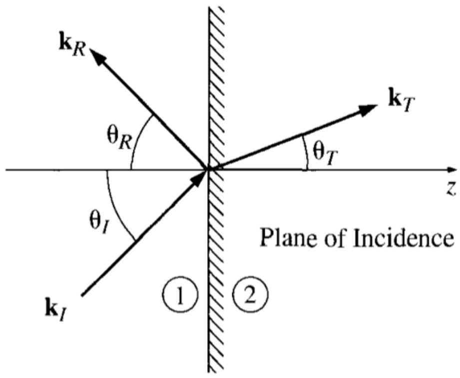
   whereas the momentum density $\mathfrak{P}$, stored in the fields, is $\mathfrak{P} = \mathbf{S}/c^2$.

a) Using the conservation of momentum, show that the pressure $P$ applied to the screen (the so-called radiation pressure) is equal to the energy density of the wave. For this purpose, time average the Poynting vector. Comment on why such an averaging is physically justified?

b) In the earth's neighborhood the electromagnetic energy flow (originating from the sun) is about $0.13\,\mathrm{W/cm^2}$. What would be the acceleration (caused by the solar radiation pressure) of the spacecraft consisting of a capsule with mass $10^5$ kg and a "solar sail" with surface density $10^{-4}\,\mathrm{g/cm^2}$ and dimensions $10\,\mathrm{km} \cdot 10\,\mathrm{km}$?

## 3. Reflection and transmission (Oral)

The space is filled with two different non-conductive media. Consider incident, reflected and transmitted monochromatic plane waves as in the figure below.




a) The frequency of each plane wave is fixed. Why and how are the wave numbers $\mathbf{k}_i$ related to each other in terms of the angles $\theta_i$, where $i \in \{I, T, R\}$?

b) In general, how are the electric and the magnetic fields related to each other at the interface? Using the results from task a) simplify these relations.

c) Show for s-polarized light ($E$-field perpendicular to the incidence plane, also referred to as Transverse Electric (TE) waves) that $1 + r = t$ where $r = E_R/E_I$ and $t = E_T/E_I$. Next, derive a relation of the form $1 - |r|^2 = c|t|^2$. Determine the constant $c$. Then, find the corresponding equations for p-polarized light ($B$-field perpendicular to the plane of incidence, $E$-field parallel to the page, also called Transverse Magnetic (TM) waves).

d) Consider a plane wave that passes from the vacuum to a medium with $n' \in \mathbb{R}$. The interface between the vacuum and the medium is perpendicular to the propagation direction. Find the reflected and the transmitted power.