Theoretische Physik III: Klassische Elektrodynamik, Exercise P

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The following exercise problem is **optional** and does not count to the mandatory 80% of the written exercise problem points to achieve the "Schein". The written points here serve as possible bonus points. Please hand in your solution in the next exercise session, after the upcoming lecture free week.

1. Demagnetization (Written) [6pt]

In this problem we investigate the effect of demagnetization, when we place a non-spherical object into a magnetic field. We are interested in a cigar shaped and discus shaped object (prolate and oblate ellipsoid of rotation, respectively). The rotation axis is z, the longer (shorter) axis is a (b), see figure. The magnetic field $\mathbf{B}$ is parallel to the z-axis.

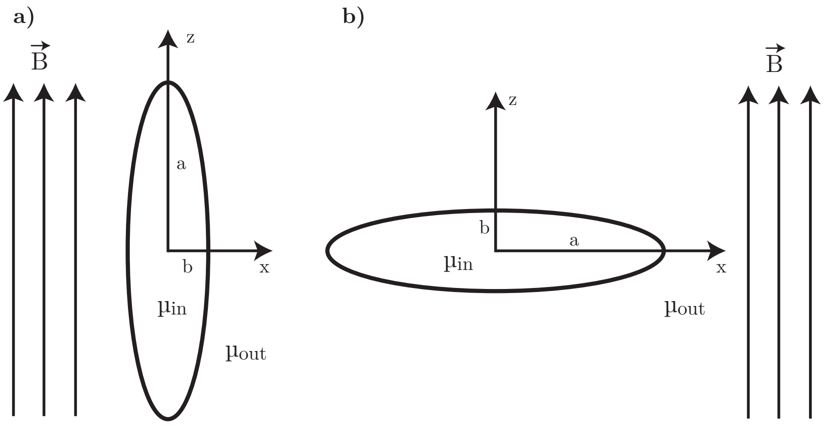

Figure 1: Sketches of the two cases **a)** prolate (cigar shaped) **b)** oblate (discus shaped) object, within a magnetic field $\mathbf{B}$.

In this scenario we consider the system without currents present, thus rot$\mathbf{H} = 0$ and div$\mathbf{H} = 0$. In addition, we are interested in a linear medium, i.e. $\mathbf{B} = \mu\,\mathbf{H}$. Hence, we can write $\mathbf{H} = -\nabla\Phi_m$. We label the fields inside the object with $\mathbf{H}_{in}$ and outside with $\mathbf{H}_{out}$.

a) Give the boundary conditions for $\mathbf{B}_i$, $\mathbf{H}_i$ and $\Phi_{m,i}$. Write down the general equation for the given ellipsoids of rotation.

b) **Prolate Case:**
In order to tackle this problem, we need a handy coordinate system. Here we

choose hyperbolic coordinates with

$$\begin{aligned} x &= c \sinh u \sin v \cos\phi \\ y &= c \sinh u \sin v \sin\phi \\ z &= c \cosh u \cos v \end{aligned} \tag{1}$$

- What is c? Give the conditions/limits for u, v, ϕ and the relations for a, b.
- Write down the general expression for the Laplace operator Δ in arbitrary coordinates.
- Transform the Laplace operator into these curvilinear coordinates and give the Laplace problem $\Delta\Phi = 0$ in these coordinates.
- Make an ansatz in order to solve the Laplace problem through separation. What are the possible solutions?
- Now find the solutions for Φ_{in} and Φ_{out} and write them down.
- Next, use the boundary conditions to determine the constants within the chosen ansatz. Then calculate all the fields.

The demagnetization factor is defined through

$$\mathbf{H}_{out} = (1 - (1-\mu)\, n)\, \mathbf{H}_{in}\,, \tag{2}$$

where $\mu = \mu_{in}/\mu_{out}$.

- Determine n by inserting the calculated fields $\mathbf{H}_{out}$ and $\mathbf{H}_{in}$.
- Rearrange this expression to receive $n = n(\epsilon)$, where ϵ is the eccentricity with $\epsilon = \frac{c}{a} = \frac{\sqrt{a^2-b^2}}{a}$.
- Investigate the limits for $n(\epsilon)$ with $\epsilon \to 0$ and $\epsilon \to 1$. For the latter limit replace $\epsilon = \sqrt{1-\eta^2}$ and let $\eta \to 0$).

c) **Oblate Case:**
Here we turn our attention to the oblate case and use as the handy coordinates

$$\begin{aligned} x &= c \cosh u \sin v \cos\phi \\ y &= c \cosh u \sin v \sin\phi \\ z &= c \sinh u \cos v \end{aligned} \tag{3}$$

Apply all the steps from the previous task to this case and determine n and its limits.

d) Why is n small for a cigar and large for a discus? Give the reasons graphically. Why is the demagnetization factor called like this?

Hint: Everything can be related to Legendre functions.