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Exercise 1: Feynman Diagrams for ϕ^4-theory (Written, 6 points)

In this exercise, we will practice writing down the analytic expressions that correspond to the Feynman diagrams. We start with the two-point Green's function

$$\langle\Omega|\,\mathcal{T}\phi(x)\phi(y)\,|\Omega\rangle\,, \tag{1}$$

where $\mathcal{T}$ refers to the time-ordered product and $|\Omega\rangle$ to the vacuum of the theory of interest. Using perturbation theory, we can expand the numerator of the solution as power series,

$$\langle 0|\,\mathcal{T}\{\phi(x)\phi(y)+\phi(x)\phi(y)\left[-i\int dt H_I(t)\right]+\cdots\}\,|0\rangle\,, \tag{2}$$

where the first term gives the free-field result, $\langle 0|\,\mathcal{T}\{\phi(x)\phi(y)\}\,|0\rangle = D_F(x-y)$.

Let us concentrate in the ϕ^4 theory, where $H_I = \int d^3x \frac{\lambda}{4!}\phi_I^4$.

a) In position space, applying Wick's theorem to the terms of λ and λ^2, draw the Feynman diagrams for all possible contractions and then write down their expressions.

b) By introducing the Fourier expansion of each propagator,

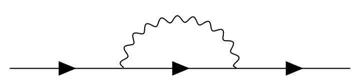
$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\epsilon}e^{-ip\cdot(x-y)}, \tag{3}$$

draw the Feynman diagrams of λ and λ^2 terms in momentum space and write down their corresponding expressions.

c) No matter whether in position or momentum space, the expression for each diagram obtained should be divided by its symmetry factor in the end. Find the symmetry factors for all the diagrams of λ and λ^2 terms.

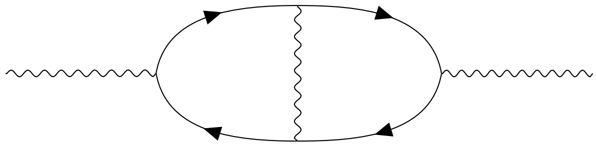
Exercise 2: Feynman Diagrams in Quantum Electrodynamics (Oral)

a) Consider the following Feynman diagram:



Label the diagram according to the Feynman rules and write down the analytical expression for this diagram (you don't have to evaluate the integrals).

b) Perform the same steps as in a) for the following Feynman diagram:



c) Show that the diagram from a) is UV divergent.
This divergence will be removed during renormalization which will be discussed later in the lecture.

d) For the two-point correlation function $\langle\Omega|\mathcal{T}\psi(x)\bar{\psi}(y)|\Omega\rangle$ draw all connected Feynman diagrams up to second order.
Derive the correct sign for the first order diagrams using Wick's theorem.
What determines the overall sign of the diagram? Can you generalize this to higher-order diagrams?