Prof. Dr. Hans Peter Büchler
Tutors: Jan Kumlin, Dr. Feiming Hu, and Dr. Daniel Huerga
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### Exercise 1: Cross Section of Two Scattering Particles (Written, 4 points)

In this exercise, we will study the cross section of two particles with $\phi^4$ theory. We start from the general formula for the differential cross section which includes two in-going particles and $n$ out-going particles,

$$d\sigma_{n\leftarrow 2} = \frac{1}{4[(p_1\cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} \int_\Delta d\tilde{p}_3 \cdots d\tilde{p}_{n+2} |\langle p_3, \cdots, p_{n+2}| \mathcal{T} |p_1, p_2\rangle|^2 \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - \cdots - p_{n+2}). \tag{1}$$

a) Show that in $\phi^4$-theory, up to lowest order, the differential cross section in the center-of-mass frame is given by

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\lambda^2}{64\pi^2 E_{\text{cm}}^2}, \tag{2}$$

where $E_{\text{cm}}$ is the center-of-mass energy.

b) Calculate the total cross section.

### Exercise 2: Rutherford scattering (Oral)

The cross section for the scattering of an electron by the Coulomb field of a nucleus can be computed, to lowest order, without quantizing the electromagnetic field. Instead, let us treat the field as a given classical potential $A_\mu(x)$. The interaction Hamiltonian is

$$H_I = \int d^3x\, e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x), \tag{3}$$

where $\psi(x)$ is the usual quantized Dirac field.

a) Show that the $T$-matrix element for electron scattering off a localized classical potential is, to lowest order,

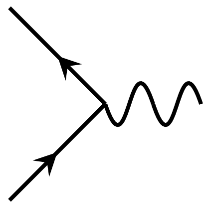
$$\langle p'| iT |p\rangle = -ie\bar{u}(p')\gamma^\mu u(p) \cdot \widetilde{A}_\mu(p' - p), \tag{4}$$

where $\widetilde{A}_\mu(q) = \int d^4x\, e^{-iqx} A_\mu(x)$ is the four-dimensional Fourier transform of $A_\mu(x)$.

b) If $\widetilde{A}_\mu(q)$ is time independent, its Fourier transform contains a delta function of energy. It is then natural to define

$$\langle p'| iT |p\rangle \equiv -i\mathcal{M} \cdot (2\pi)\delta(E_f - E_i), \tag{5}$$

where $E_i$ and $E_f$ are the initial and final energies of the particle, and to adopt a new Feynman rule for computing $\mathcal{M}$:



$$= -ie\gamma^\mu \widetilde{A}_\mu(\mathbf{q}) \tag{6}$$

where $\widetilde{A}(\mathbf{q})$ is the three-dimensional Fourier transform of $A_\mu(x)$. Given this definition of $\mathcal{M}$, show that the cross section for scattering off a time-dependent localized potential is

$$d\sigma = \frac{1}{v_i}\frac{1}{2E_i}\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}|\mathcal{M}(p_i \to p_f)|^2(2\pi)\delta(E_f - E_i), \tag{7}$$

where $v_i$ is the particle's initial velocity. Integrate over $|p_f|$ to find a simple expression for $d\sigma/d\Omega$.

c) Let us restrict to the case of electron scattering off a Coulomb potential

$$A^0 = \frac{Ze}{4\pi r}. \tag{8}$$

Working in the non-relativistic limit, derive the Rutherford formula,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Z^2}{4m^2 v^4 \sin^4(\theta/2)}. \tag{9}$$