Exercise 3: (4 points)

Using the definition of functional derivatives show explicitly that

$$\left.\frac{\delta^2 \mathrm{e}^{W[J]}}{\delta J(\boldsymbol{x}_1)\delta J(\boldsymbol{x}_2)}\right|_{J=0} = G_0(\boldsymbol{x}_1, \boldsymbol{x}_2) \; , \tag{1}$$

for

$$W[J] = \frac{1}{2}\int \mathrm{d}^d x \, \mathrm{d}^d x' \; J(\boldsymbol{x}) G_0(\boldsymbol{x}, \boldsymbol{x}') J(\boldsymbol{x}') \; . \tag{2}$$

Exercise 4: Diagrammatic analysis of the ϕ^3-theory (10 points)

The action for the ϕ^3-theory in d-dimensions is given by

$$S = \int \mathrm{d}^d r \left[\frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2} m_0^2 \phi^2 + \frac{\lambda}{3!}\phi^3\right] \tag{3}$$

i) Write down all the diagrams that contribute to the two-point function $\langle\phi(k_1)\phi(k_2)\rangle$ and the four-point function $\langle\phi(k_1)\phi(k_2)\phi(k_3)\phi(k_4)\rangle$ up to second order in λ (This means also including disconnected diagrams and diagrams with vacuum insertions).

ii) Write down the vacuum diagrams up to the same order and show how some of the diagrams in the two-point and four-point functions is cancelled by the normalization.

iii) For each diagram, use the Feynman rules to express its value in terms of momentum integrals.

iv) Identify which diagrams are connected and which are not.

v) Identify which of the diagrams are one-particle irreducible (1PI).