Perturbation theory

Consider a ϕ^4-theory in d-dimensional space for the O(n) case, with $n > 1$, whose action is given by

$$S = \int \mathrm{d}^d x \, \left\{ \frac{1}{2} \frac{\partial \phi^\alpha}{\partial x^\mu} \frac{\partial \phi^\alpha}{\partial x^\mu} + \frac{1}{2} m_0^2 \phi^\alpha \phi^\alpha + \frac{\lambda_0}{4!} \left[\phi^\alpha \phi^\alpha \right]^2 - J^\alpha \phi^\alpha \right\} ,$$

where $\mu = 1, \ldots, d$, $\alpha = 1, \ldots, n$, and summation over repeated indices is understood. The correlation functions below should be considered in the limit $J \to 0$.

Exercise 4: Free-theory (5 points)

i) Calculate the 2-point correlation function

$$< \phi^\alpha(\boldsymbol{x}) \phi^\beta(\boldsymbol{x}') >$$

for the free part (apply explicitely functional derivation). (2 points)

ii) For a spin-problem one generally calculates the following correlation function

$$< \boldsymbol{S}(\boldsymbol{x}) \cdot \boldsymbol{S}(\boldsymbol{x}') > \; \longrightarrow \; < \phi^\alpha(\boldsymbol{x}) \phi^\alpha(\boldsymbol{x}') > \; .$$

Which is the result for the free-theory? (1 point)

iii) Calculate the following correlation function for the free-theory

$$< \phi^\alpha(\boldsymbol{x}_1) \, \phi^\beta(\boldsymbol{x}_2) \, \phi^\gamma(\boldsymbol{x}_3) \, \phi^\delta(\boldsymbol{x}_4) > \; .$$

On the basis of the obtained result, propose a general rule. (2 points)

Hint: A functional derivative of a functional $F\left[\phi(\boldsymbol{x})\right]$ is defined as

$$\int \mathrm{d}^d x \frac{\delta F[\phi]}{\delta \phi(\boldsymbol{x})} f(\boldsymbol{x}) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left\{ F\left[\phi + \epsilon f\right] - F\left[\phi\right] \right\} ,$$

where f is an arbitrary differentiable function.

Exercise 5: Interacting theory (3 points)

Calculate the 2-point correlation function

$$< \phi^\alpha(\boldsymbol{x}) \phi^\alpha(\boldsymbol{x}') >$$

up to $\mathcal{O}(\lambda_0^2)$. Show the corresponding diagrams.