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Exercise 1: Crystal Structure (Oral)

a) The primitive vectors for a *body-centered cubic* (bcc) lattice are

$$\boldsymbol{a}_1 = a\boldsymbol{x}, \tag{1}$$
$$\boldsymbol{a}_2 = a\boldsymbol{y}, \tag{2}$$
$$\boldsymbol{a}_3 = \frac{a}{2}(\boldsymbol{x}+\boldsymbol{y}+\boldsymbol{z}), \tag{3}$$

or in a more symmetric set

$$\boldsymbol{a}_1 = \frac{a}{2}(-\boldsymbol{x}+\boldsymbol{y}+\boldsymbol{z}), \tag{4}$$
$$\boldsymbol{a}_2 = \frac{a}{2}(\boldsymbol{x}-\boldsymbol{y}+\boldsymbol{z}), \tag{5}$$
$$\boldsymbol{a}_3 = \frac{a}{2}(\boldsymbol{x}+\boldsymbol{y}-\boldsymbol{z}), \tag{6}$$

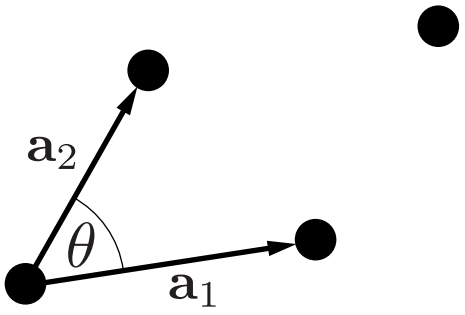
Draw the lattice, find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

b) The primitive vectors for a *face-centered cubic* (fcc) lattice are

$$\boldsymbol{a}_1 = \frac{a}{2}(\boldsymbol{y}+\boldsymbol{z}), \tag{7}$$
$$\boldsymbol{a}_2 = \frac{a}{2}(\boldsymbol{x}+\boldsymbol{z}), \tag{8}$$
$$\boldsymbol{a}_3 = \frac{a}{2}(\boldsymbol{x}+\boldsymbol{y}), \tag{9}$$

Draw the lattice, find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

c) Graphically construct the Wigner-Seitz cell and the reciprocal lattice of the two-dimensional oblique lattice with basis vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ shown in the following sketch:



Exercise 2: Tetragonal Symmetry (Oral)

Show that if one stretches a fcc lattice along one of its lattice vectors, the resulting lattice is equivalent to a tetragonal body-centered lattice. So the point group with the tetragonal symmetry has two Bravais lattices: simple tetragonal and body-centered tetragonal, whereas the point group with cubic symmetry has three Bravais lattices: sc, bcc and fcc.

Exercise 3: The Brilluoin Zone (Written)

Show that the volume of the elementary cell Ω and the volume of the Brillouin zone Ω_B are connected by the following relation:

$$\Omega_B = \frac{(2\pi)^3}{\Omega} \tag{10}$$

Exercise 4: Fourier Transformations (Written)

a) Calculate the Fourier coefficients for a function $f(x) = c$, c some number, x defined in some one-dimensional interval.

b) Calculate explicitely the Fourier coefficients for a function $f(x) = \exp\left(i\, 2\pi n \frac{x}{a}\right)$ with some integer n.