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Exercise 1: Dielectric function in random-phase approximation (Oral)

In the random-phase approximation (RPA), the dielectric function is calculated by resummation of the most diverging diagrams in each order of the diagrammatic expansion and is given by

$$\varepsilon_{\mathrm{RPA}}(\boldsymbol{q},\omega) = 1 - V_{\boldsymbol{q}}\chi^0(\boldsymbol{q},\omega) \tag{1}$$

with the non-interacting response function χ^0. For $T=0$, the dielectric function in RPA, $\varepsilon_{\mathrm{RPA}} = \varepsilon'_{\mathrm{RPA}} + i\varepsilon''_{\mathrm{RPA}}$ can be calculated analytically and is given by

$$\begin{aligned}
\varepsilon'_{\mathrm{RPA}} &= 1 + \frac{1}{\lambda_{TF}^2 q^2}\left\{\frac{1}{2} + \frac{p_F}{4q}\left[\left(\frac{(\omega+q^2/2m)^2}{(qv_F^0)^2} - 1\right)\log\left|\frac{\omega - qv_F^0 + q^2/2m}{\omega + qv_F^0 + q^2/2m}\right|\right.\right. && (2)\\
&\quad \left.\left. - \left(\frac{(\omega-q^2/2m)^2}{(qv_F^0)^2} - 1\right)\log\left|\frac{\omega - qv_F^0 - q^2/2m}{\omega + qv_F^0 - q^2/2m}\right|\right]\right\} && (3)
\end{aligned}$$

$$\varepsilon''_{\mathrm{RPA}} = \frac{\pi/2}{\lambda_{TF}^2 q^2}\begin{cases}\omega/qv_F^0 & 0<\omega<qv_F^0 - q^2/2m\,,\\ \frac{p_F}{q}\left[1 - \frac{(\omega-q^2/2m)^2}{(qv_F^0)^2}\right] & |\omega - qv_F^0| < q^2/2m\,,\\ 0 & qv_F^0 + q^2/2m \le \omega\,,\end{cases} \tag{4}$$

where $\lambda_{TF} = \sqrt{\frac{\epsilon_F}{6\pi n e^2}}$ is the Thomas-Fermi screening length.

a) Calculate the limits $\varepsilon(0,\omega)$ and $\varepsilon(\boldsymbol{q}\to 0, 0)$.

b) Show that the plasma dispersion is given by

$$\omega_q = \omega_p\left[1 + \frac{3}{10}\left(\frac{qv_F^0}{\omega_p}\right)^2\right]. \tag{5}$$

Exercise 2: Meissner-Ochsenfeld effect (Oral)

From the lecture you know that in a superconductor, the transverse current density is proportional to the transverse component of the vector potential:

$$\boldsymbol{j}_\perp = -\frac{e^2 n_s}{mc}\boldsymbol{A}_\perp \tag{6}$$

with the superfluid density n_s. Show that this leads to the following differential equation for the magnetic field

$$\nabla^2\boldsymbol{B} = \frac{1}{\lambda^2}\boldsymbol{B} \tag{7}$$

and determine λ. In order to solve this equation, assume a semi-infinite superconductor for $x > 0$ and a constant magnetic field $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{z}}$ parallel to the surface. Show that inside the superconductor, the field decays exponentially with the so-called *London penetration length* λ.