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Exercise 1: Hydrogen atom – lowest states (Oral, 3 Pts.)

Consider the wave functions of the Hydrogen atom as derived in the Lecture, $\psi_{n,\ell,m}$.

a) Write explicitly the $1s$, $2s$, and $2p$ wave functions, which correspond to the set of quantum numbers $\{n,\ell,m\} = \{1,0,0\}$, $\{2,0,0\}$, and $\{2,1,0\}$, respectively.

b) Using the previously derived expressions, show explicitly that they are orthonormal.

c) Determine the expectation value of the radial components r, r^2, p and p^2 in the ground state (i.e. the $1s$ state). Determine the velocity and show that it scales as the fine structure constant, α.

Exercise 2: Hydrogen atom – coherent states (Written, 4 Pts.)

Let us consider states of the Hydrogen atom with quantum numbers $\ell = n-1$, and $m = \pm\ell$. In the limit $n \to \infty$ they correspond to circular orbitals.

a) Write explicitly the wave function $\psi_{n,\ell,m}(r,\theta,\varphi)$ for $\ell = m = n-1$.

b) How does $\langle r \rangle$ scale with the principal quantum number n for the wave function $\psi_{n,n-1,n-1}(r,\theta,\varphi)$?

c) Calculate the relative variance $\Delta r/\langle r \rangle$ as a function of the principal quantum number n with $\psi_{n,n-1,n-1}(r,\theta,\varphi)$.

d) Plot the probability density $|\psi_{n,n-1,n-1}|^2$ in the plane of motion (plane of the trajectory) and its orthogonal direction when the principal quantum number takes the values $n = 5, 20, 50$, and 100.