Prof. Dr. Hans Peter Büchler
Institut für Theoretische Physik III, Universität Stuttgart

24. Januar 2016
WS 2016/17

Exercise 1: Stark Effect in harmonic Oscillator (Oral, 4 pts.)

The Hamiltonian operator for a 1-dimensional harmonic oscillator in a homogeneous electric field E reads

$$H = \frac{1}{2}(P^2 + Q^2) + eEQ \tag{1}$$

Consider the second term in the Hamiltonian as perturbation of the free harmonic oscillator, with $H_1 := Q$ and $\lambda := eE$.

a) Compute the perturbed eigenfunctions and eigenvectors up to order 3 in λ.

b) Using the substitution $Y = Q + \lambda$, find the exact eigenvalues of the Hamiltonian and compare your result with (a).

Exercise 2: Perturbation in 2-level System (Oral, 4 pts.)

The unperturbed Hamiltonian operator for a 2-level system reads in its own eigenbasis

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \tag{2}$$

Consider the perturbed Hamiltonian operator of the form

$$H(\lambda) := H_0 + \lambda \boldsymbol{e} \cdot \boldsymbol{\sigma}. \tag{3}$$

(with σ_i the Pauli matrices and $\mathbf{e} := (e_x, e_y, e_z)$ a general vector)

a) Compute the eigenfunctions and eigenenergies up to 2nd order in λ in the non-degenerate case ($E_1 \neq E_2$).

b) In the degenerate case ($E_1 = E_2$), find a linear combination of unperturbed eigenvectors which diagonalizes the perturbed Hamiltonian. How does the 1st order correction to the eigenenergies read?

c) Solve exactly the Schrödinger equation

$$H(\lambda)\psi(\lambda) = E(\lambda)\psi(\lambda) \tag{4}$$

and compare your result with (a) and (b).

Exercise 3: Projection Operator (Written, 3 bonus pts.(*))

Define the Projection operator P as $P^2 = P$ with $P^\dagger = P$.

a) Show that the eigenvalues are 0 and 1.

b) Given the subspace generated by $|\psi_i\rangle$ $(i = 1, ..., N)$. Show that $P = \sum_i |\psi_i\rangle \langle\psi_i|$ is a Projection operator for the subspace.

c) Given the initial state $|\psi\rangle$, consider the measurement of P (as given in (b)) with observed value 1. What is the state after the measurement?