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Exercise 1: Planck's radiation law (Written, 3p)

Based on the tips given in the lecture, derive Planck's formula for the spectral energy density and total energy density of the radiation emitted by a black body at temperature T.

Exercise 2: Einstein's approach to absorption and emission of photons (Oral)

Consider a system consisting of two-level atoms with energy difference between the two states equal to $E_2 - E_1 = \hbar\omega$. As the atom absorbs or emits a photon, it changes its internal state.

a) First, let's consider spontaneous emission. Only atoms in the excited state can emit photons. The number of excited atoms can be denoted as N_2. Write the rate equation for the emission process by introducing Einstein's coefficient A_{21}. What is the solution to this equation?

b) Now let's add stimulated processes. For this we introduce Einstein's B_{12} and B_{21} coefficients. Also, the spectral energy density of the radiation field $u(\omega)$ is needed. The rates are proportional to $Bu(\omega)$. Write the rate equations for both $|1\rangle \to |2\rangle$ (absorption fo radiation) and $|2\rangle \to |1\rangle$ (stimulated emission) processes. The latter is especially important for the lasers to work. What is the special property of stimulated emission?

c) Write the total rate equations for all the processes. Assume Boltzmann distribution for N_2/N_1 and Planck's formula for the energy density. We know that we should have $dN_1/dt = -dN_2/dt$ (particle number conservation). What is the condition for the Einstein's coefficients that we get from this?

Exercise 3: Coherent light pulses (Oral)

The creation operator of a single photon in mode $\mathbf{k}$ (we omit polarization here) is written as $a_{\mathbf{k}}^\dagger$. In free space, the modes are plain waves. We can now build from them a superposition state (in the continuum limit)

$$|\psi\rangle = \psi^\dagger |0\rangle = \int \frac{d^3k}{(2\pi)^3} f(\mathbf{k}) a_{\mathbf{k}}^\dagger |0\rangle .$$

a) What is the condition under which this state is normalized? Calculate the commutator $\left[\psi, \psi^\dagger\right]$. Is ψ a proper bosonic operator? Assume a gaussian shape of the wave packet $f(\mathbf{k}) \propto e^{-\alpha(\mathbf{k}-\mathbf{k}_0)^2/2}$. How does it evolve in time? Does it spread?

b) Calculate the mean values of the electric field $\langle E \rangle$ and its square $\langle E^2 \rangle$ on a number state $\frac{(a_k^\dagger)^n}{\sqrt{n!}} |0\rangle$. Does the result for large photon numbers correspond to what one expects from a classical system?

c) Now take a coherent state built from $\psi^\dagger$. Calculate the mean value of the electric field $\langle E(\mathbf{r}, t) \rangle$ for this state, the mean value of E^2 and the fluctuations $\Delta^2 E = \langle E^2 \rangle - \langle E \rangle^2$. How does $\Delta E / \langle E \rangle$ behave with increasing mean number of photons?