Exercise 1 - Crystal Structure (3 points)

(a) The primitive vectors for a *body center cubic* (bcc) lattice are

$$\begin{aligned}\vec{a}_1 &= a\vec{x},\\ \vec{a}_2 &= a\vec{y},\\ \vec{a}_3 &= \frac{a}{2}(\vec{x}+\vec{y}+\vec{z}).\end{aligned}$$

or, in a more symmetric set

$$\begin{aligned}\vec{a}_1 &= \frac{a}{2}(-\vec{x}+\vec{y}+\vec{z}),\\ \vec{a}_2 &= \frac{a}{2}(\vec{x}-\vec{y}+\vec{z}),\\ \vec{a}_3 &= \frac{a}{2}(\vec{x}+\vec{y}-\vec{z}).\end{aligned}$$

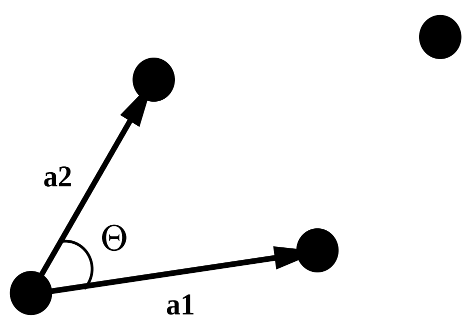
1. Make a drawing of this lattice,
2. Find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

(b) The primitive vectors for a *face center cubic* (fcc) lattice are

$$\begin{aligned}\vec{a}_1 &= \frac{a}{2}(\vec{y}+\vec{z}),\\ \vec{a}_2 &= \frac{a}{2}(\vec{x}+\vec{z}),\\ \vec{a}_3 &= \frac{a}{2}(\vec{x}+\vec{y}).\end{aligned}$$

1. Make a drawing of this lattice,
2. Find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

(c) Graphically construct the Wigner-Seitz cell and the reciprocal lattice of the two dimensional oblique lattice with basis vectors $\vec{a}1$ and $\vec{a}2$ shown in the following sketch:



Exercise 2 - The Brillouin Zone (1 points)

Show that the volume of the elementary cell Ω and the volume of the Brillouin Zone Ω_B are connected by the following relation:

$$\Omega_B = \frac{(2\pi)^3}{\Omega} \tag{1}$$

Exercise 3 - Tetragonal symmetry (2 points)

Show that if one streches a fcc lattice along one of its lattice vectors, the resulting lattice is equivalent to a tetragonal body centered lattice. So the point group with the tetragonal symmetry has two Bravais lattices: simple tetragonal and body centered tetragonal, whereas the point group with cubic symmetry has three Bravais lattices: sc, bcc and fcc.

Exercise 4 - Fourier transformations (2 points)

1. Calculate the Fourier coefficients for a function $f(x) = c$, c some number, x defined in some one-dimensional interval.
2. Calculate explicitely the Fourier coefficients for a function $f(x) = \exp\big(i\,p\frac{2\pi}{a}x\big)$, with p some integer.

Solutions due on the 15th of April, 2013