Exercise 1 - Crystal Structure

(a) The primitive vectors for a *body center cubic* (bcc) lattice are

\[
\begin{align*}
\vec{a}_1 &= a\vec{x}, \\
\vec{a}_2 &= a\vec{y}, \\
\vec{a}_3 &= \frac{a}{2}(\vec{x} + \vec{y} + \vec{z}).
\end{align*}
\]

or, in a more symmetric set

\[
\begin{align*}
\tilde{a}_1 &= \frac{a}{2}(-\vec{x} + \vec{y} + \vec{z}), \\
\tilde{a}_2 &= \frac{a}{2}(\vec{x} - \vec{y} + \vec{z}), \\
\tilde{a}_3 &= \frac{a}{2}(\vec{x} + \vec{y} - \vec{z}).
\end{align*}
\]

1. Make a drawing of this lattice,
2. Find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

(b) The primitive vectors for a *face center cubic* (fcc) lattice are

\[
\begin{align*}
\vec{a}_1 &= \frac{a}{2}(\vec{y} + \vec{z}), \\
\vec{a}_2 &= \frac{a}{2}(\vec{x} + \vec{z}), \\
\vec{a}_3 &= \frac{a}{2}(\vec{x} + \vec{y}).
\end{align*}
\]

1. Make a drawing of this lattice,
2. Find the reciprocal lattice vectors and make a drawing of the reciprocal lattice.

(c) Graphically construct the Wigner-Seitz cell and the reciprocal lattice of the two dimensional oblique lattice with basis vectors \(\vec{a}_1\) and \(\vec{a}_2\) shown in the following sketch:
Exercise 2 - The Brillouin Zone

Show that the volume of the elementary cell $\Omega$ and the volume of the Brillouin Zone $\Omega_B$ are connected by the following relation:

$$\Omega_B = \frac{(2\pi)^3}{\Omega} \quad (1)$$

Exercise 3 - Tetragonal symmetry

Show that if one stretches a fcc lattice along one of its lattice vectors, the resulting lattice is equivalent to a tetragonal body centered lattice. So the point group with the tetragonal symmetry has two Bravais lattices: simple tetragonal and body centered tetragonal, whereas the point group with cubic symmetry has three Bravais lattices: sc, bcc and fcc.

Exercise 4 - Fourier transformations

1. Calculate the Fourier coefficients for a function $f(x) = c$, $c$ some number, $x$ defined in some one-dimensional interval.

2. Calculate explicitly the Fourier coefficients for a function $f(x) = \exp(i p \frac{2\pi}{a} x)$, with $p$ some integer.

Solutions due on the 15th of April, 2013