Exercise 1 - Theorem of residues (3 points)

Applying the theorem of residues,

a) calculate the Fourier transform of

$$f(x) = \frac{1}{x^2 + a^2} \ , \tag{1}$$

where x is a one-dimensional real variable and a is a real number.

b) and verify the integral representation of the θ function

$$\theta(t - t') = \lim_{\eta \to 0^+} - \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{e^{-i\omega(t-t')}}{\omega + i\eta} \ . \tag{2}$$

Exercise 2 - Ground state properties of the Fermi gas (2 points)

In the lecture you derived the ground state energy of the Fermi-gas as

$$E_0 := \langle \Psi_0 | H | \Psi_0 \rangle = \frac{3}{5} N E_F \ . \tag{3}$$

Show explicitely that the relation

$$E_0(N + 1) - E_0(N) = \mu \ , \tag{4}$$

is fulfilled for a Fermi gas at $T = 0$. Note that at $T = 0$, the chemical potential μ is just the Fermi energy.

Solutions due on: 24 June, 2013