Exercise 1 - Superconductivity and Ginzburg-Landau theory (4 points)

The Ginzburg-Landau equation in one dimension without magnetic field is given by

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) - a\frac{T_c - T}{T_c}\psi(x) + b\,\psi^3(x) = 0\ . \tag{1}$$

We assume that half of the space $x > 0$ is occupied by a superconductor, and the other half $x < 0$ by a normal conductor. The phase of the order parameter was chosen such that $\psi(x)$ is real.

(a) Rewrite the Ginzburg-Landau equation in dimensionless form, by means of an appropriate rescaling $\psi(x) \to \tilde{\psi}(x)$. The resulting differential equation will depend on a single constant $\xi(T)$ only, which is interpreted as the coherence length. Derive this constant and show that the solution reads

$$\tilde{\psi}(x) = \tanh\frac{x}{\sqrt{2}\xi}\ ,\ x > 0 \tag{2}$$

We apply an external magnetic field $H \lesssim H_c$, where H_c denotes the critical magnetic field. The Ginzburg-Landau equation then reads

$$-\frac{1}{\kappa^2}\frac{\partial^2}{\partial x^2}\tilde{\psi}(x) + \tilde{A}^2(x)\tilde{\psi}(x) - \tilde{\psi}(x) + \tilde{\psi}^3(x) = 0\ ,\ \text{where} \tag{3}$$

where $\tilde{A}(x) = \frac{A(x)}{\sqrt{2}\lambda H_c}$ is the dimensionless magnetic potential.

(b) Calculate the parameter $\kappa = \lambda(T)/\xi(T)$, where $\lambda(T)$ is the London penetration depth and discuss the solution of (3) in the region where

- $x >> 0$, so that $\tilde{\psi}(x) \approx 1$ and the magnetic field is completely surpressed.
- $x \approx 0$, where $\tilde{\psi}(x) \approx 0$ and the magnetic potential $\tilde{A} \approx x$.

Distinguish the cases $\xi \gg \lambda$ and $\xi \ll \lambda$. Sketch the behaviour of $\tilde{\psi}(x)$ and the magnetic field $B(x)$ which is determined through London's equation. How do the two cases correspond to the different types of superconductors?

Solutions due on: 15 July, 2013