Exercise 1 - Superconductivity and Ginzburg-Landau theory

The Ginzburg-Landau equation in one dimension without magnetic field is given by

\[-\hbar^2 \frac{d^2}{dx^2} \psi(x) - a \frac{T_c - T}{T_c} \psi(x) + b \psi^3(x) = 0.\]

(1)

We assume that half of the space \( x > 0 \) is occupied by a superconductor, and the other half \( x < 0 \) by a normal conductor. The phase of the order parameter was chosen such that \( \psi(x) \) is real.

(a) Rewrite the Ginzburg-Landau equation in dimensionless form, by means of an appropriate rescaling \( \psi(x) \rightarrow \tilde{\psi}(x) \). The resulting differential equation will depend on a single constant \( \xi(T) \) only, which is interpreted as the coherence length. Derive this constant and show that the solution reads

\[ \tilde{\psi}(x) = \tanh \left( \frac{x}{\sqrt{2}\xi} \right), \quad x > 0 \]

(2)

We apply an external magnetic field \( H \lesssim H_c \), where \( H_c \) denotes the critical magnetic field. The Ginzburg-Landau equation then reads

\[-\frac{1}{\kappa^2} \frac{\partial^2}{\partial x^2} \tilde{\psi}(x) + \tilde{A}^2(x) \tilde{\psi}(x) - \tilde{\psi}(x) + \tilde{\psi}^3(x) = 0, \]

where \( \tilde{A}(x) = \frac{A(x)}{\sqrt{2\lambda H_c}} \) is the dimensionless magnetic potential.

(b) Calculate the parameter \( \kappa = \lambda(T)/\xi(T) \), where \( \lambda(T) \) is the London penetration depth and discuss the solution of (3) in the region where

- \( x >> 0 \), so that \( \tilde{\psi}(x) \approx 1 \) and the magnetic field is completely suppressed.
- \( x \approx 0 \), where \( \tilde{\psi}(x) \approx 0 \) and the magnetic potential \( \tilde{A} \approx x \).

Distinguish the cases \( \xi \gg \lambda \) and \( \xi \ll \lambda \). Sketch the behaviour of \( \tilde{\psi}(x) \) and the magnetic field \( B(x) \) which is determined through London’s equation. How do the two cases correspond to the different types of superconductors?

Solutions due on: 15 July, 2013