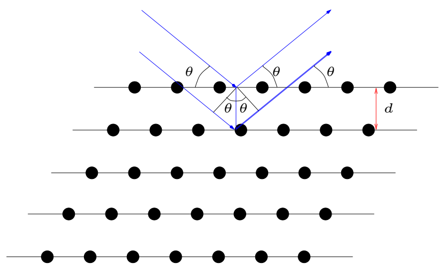
Exercise 1 - Bragg scattering with electrons (1 points)

Consider a low-energy beam of electrons, fired on a single crystal as was done in the experiment by Davisson and Germer in 1927. The electrons will be reflected off the surface, as shown in the figure, leading to an angle dependent intensity of scattered electrons. Show that this intensity has a maximum when the condition

$$\frac{nh}{\sqrt{2m_e E_e}} = 2d \sin\theta \tag{1}$$

is fulfilled, where m_e and E_e are the mass and the energy of the electron respectively, and d is the distance between two planes of atoms.



Exercise 2 - Electronic Density of States (3 points)

Consider the following tight binding dispersion

$$E(\mathbf{k}) = -2t \sum_{i=1}^{d} \cos(k_i a), \tag{2}$$

where t is the hopping integral and a is the lattice spacing.

1. Calculate the density of states (DOS) for $d = 1$ and discuss the algebraic form of the van Hove singularities at $E = \pm 2t$.

2. Calculate the DOS in $d = 3$. *Hint*: Using the tight binding dispersion in Eq. (2), calculate the number of possible van Hove points in $d = 3$, and then use a parabolic approximation for the dispersion around such points and calculate the DOS.

   Observe that the DOS is free of van Hove singularities in the limit $d \to \infty$. In fact, if we consider the sum over **k** *points in the DOS as a sum over randomly chosen numbers $-2t\cos(k_i a)$, then the* **central limit theorem** *tells us that in the limit $d \to \infty$ the DOS becomes a Gaussian, thus free of singularities.*

Solutions due on the 6th of May, 2013