Bose-Einstein condensation of cold gases in traps

Matthias Wenzel

Hauptseminar: Physik der kalten Gase

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Introduction

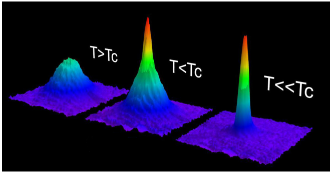

Figure : Density distribution of Sodium atoms with $T_c = 2\,\mu\mathrm{K}$. From: Nobel lecture of Wolfgang Ketterle (2001)

1995	Rb Na Li
1998	H
2001	K
2002	Cs
2004	Cr
2011	Dy

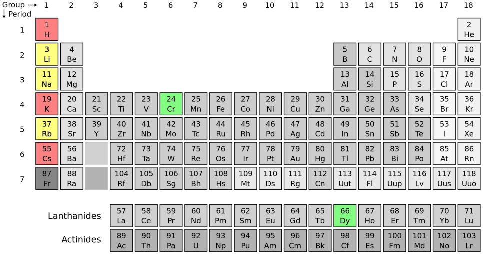

Figure : Time line and periodic table of BEC.

Outline

1. Ideal bose gas
   - Harmonic potential
   - Critical temperature T_c
   - Density distribution
2. Experimental techniques
   - Typical setup
   - Radiation forces on atoms
   - Laser cooling
   - Evaporative cooling
3. Weakly interacting bose gas
   - Thomas-Fermi approximation
   - Healing length

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Harmonic potential

Isotropic harmonic potential

$$V_{\text{ho}}(\mathbf{r}) = \frac{1}{2} m\omega_{\text{ho}}^2 \mathbf{r}^2$$

Oscillator length

$$a_{\text{ho}} = \sqrt{\frac{\hbar}{m\omega_{\text{ho}}}} \approx 5\,\mu\text{m}$$

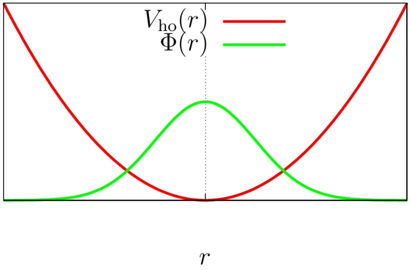

Figure : Potential and ground state of a harmonic oscillator.

Ideal bose gas

Hamiltonian

$$H = \sum_i H_i = \sum_i \left[-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\text{ho}}(\mathbf{r}_i) \right]$$

Single-particle energy and ground state $(n_x = n_y = n_z = 0)$

$$E = \hbar \omega_{\text{ho}} \left(n_x + n_y + n_z + \frac{3}{2} \right)$$

$$\Phi(\mathbf{r}) = \left(\frac{m\,\omega_{\text{ho}}}{\pi\,\hbar} \right)^{3/4} \exp\left[-\frac{m}{2\hbar} \omega_{\text{ho}}\, r^2 \right] = \left(\frac{1}{\sqrt{\pi}\, a_{\text{ho}}} \right)^{3/2} \exp\left[-\frac{r^2}{2a_{\text{ho}}^2} \right]$$

N-particle wave function and density distribution

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \prod_i \Phi(\mathbf{r}_i) \qquad\qquad n(\mathbf{r}) = N\, |\Phi(\mathbf{r})|^2$$

Blackboard: Density of states

Density of states

Critical temperature T_c

Bose distribution and density of states

$$f(E,T) = \frac{1}{e^{(E-\mu)/kT} - 1} \qquad\qquad g(E) = \frac{1}{2(\hbar\omega_{\mathrm{ho}})^3} E^2$$

Particle number in excited states, zero-point energy neglected

$$N_{\mathrm{ex}} = \int_0^\infty \mathrm{d}E\, g(E) f(E,T)$$

All particles in excited states at T_c

$$\begin{aligned}
N &= N_{\mathrm{ex}}(T = T_c, \mu = 0) = \int_0^\infty \mathrm{d}E\, \frac{g(E)}{e^{E/kT_c} - 1} \\
&= \frac{1}{2}\left(\frac{kT_c}{\hbar\omega_{\mathrm{ho}}}\right)^3 \int_0^\infty \mathrm{d}z\, \frac{z^2}{e^z - 1} = \frac{1}{2}\left(\frac{kT_c}{\hbar\omega_{\mathrm{ho}}}\right)^3 \Gamma(3)\zeta(3) \\
&= \zeta(3)\left(\frac{kT_c}{\hbar\omega_{\mathrm{ho}}}\right)^3
\end{aligned}$$

Critical temperature T_c

In a harmonic potential

$$kT_c \approx 0.94\,\hbar\omega_{\mathrm{ho}} N^{1/3}$$

... compared to a box

$$kT_c \approx 3.31 \frac{\hbar^2 n^{2/3}}{m}$$

Typical values: $\omega_{\mathrm{ho}} = 2\pi \cdot 100\,\mathrm{Hz}$ and $N = 10^5$

$$T_c \approx 500\,\mathrm{nK}$$

$$T_c \approx 50\,\mu\mathrm{K}$$

Condensate fraction

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^3$$

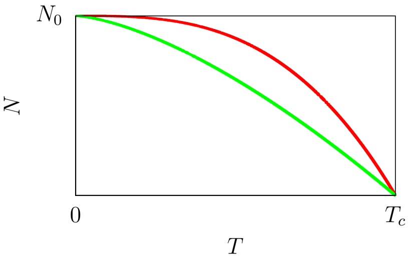

Figure : Condensate fraction of a harmonic (red) and a box potential (green).

Density distribution

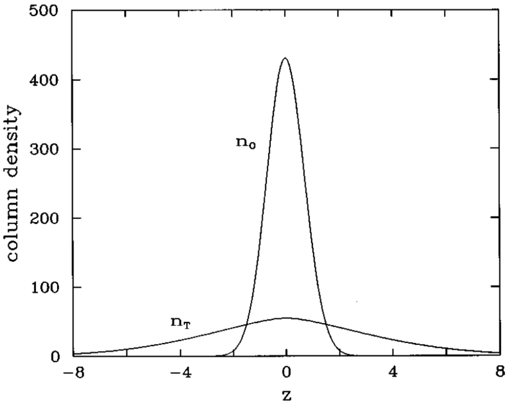

Figure : Noninteracting bosons in a spherical trap at temperature $T = 0.9\,T_c$, Length z in units of $a_{\rm ho}$. From: Dalfovo et. al. (1999)

Density distribution

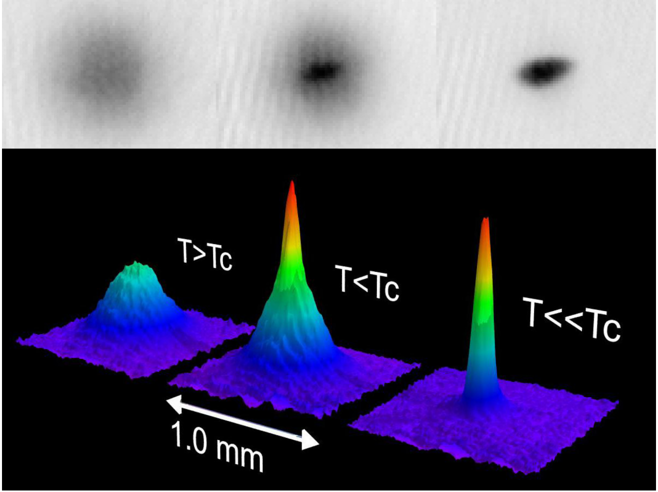

Figure : Density distribution of Sodium atoms with $T_c = 2\,\mu\mathrm{K}$. From: Nobel lecture of Wolfgang Ketterle (2001)

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Typical setup

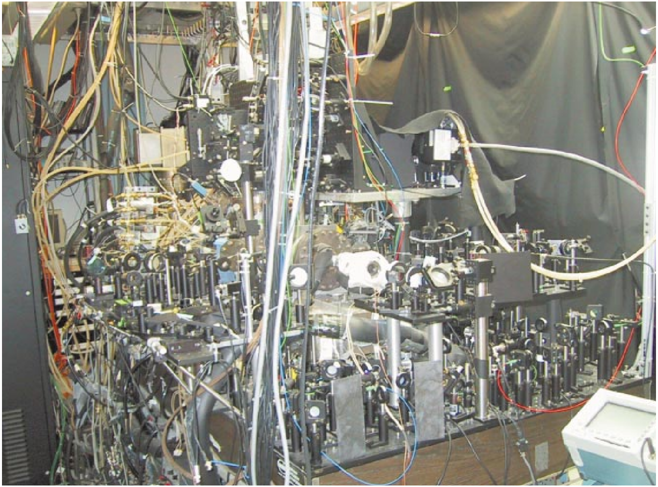
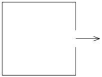
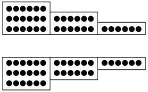
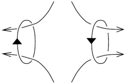
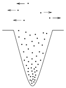


Figure : Setup of a BEC experiment. From: Nobel lecture of Wolfgang Ketterle (2001)

Typical setup

Oven		$600\ \mathrm{K}$	$800\ \mathrm{m/s}$
Zeeman slower		$1\ \mathrm{K}$	$30\ \mathrm{m/s}$
Magneto-optical trap		$100\ \mu\mathrm{K}$	10^9 atoms
Evaporative cooling in a magnetic trap		$< T_C$	10^5 atoms

Dipole force

Stark effect

$$V_S = -\frac{1}{2}\alpha'(\omega)\left\langle E^2 \right\rangle_t$$

Dipole force

$$\begin{aligned}\mathbf{F}_{\text{dip}} &= -\nabla V_S(\mathbf{r}) \\ &= \frac{1}{2}\alpha'(\omega)\,\nabla\left\langle E^2 \right\rangle_t\end{aligned}$$

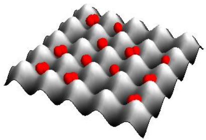


Figure : Atoms in an optical lattice potential. From: Wikipedia - Optisches Gitter

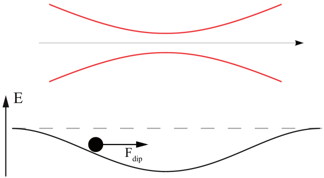


Figure : Laser beam profile and potential of an atom in this beam. From: Diploma thesis at PI5

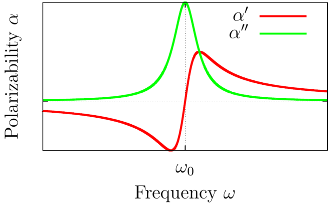


Figure : Real (red) and imaginary part (green) of the polarizability over frequency.

Scattering force

Photon momentum

$$\mathbf{p} = \hbar \mathbf{q}$$

Scattering force

$$\mathbf{F}_\mathrm{s}(\omega - \omega_0) = \dot{\mathbf{p}} = \hbar \mathbf{q}\, \Gamma_\mathrm{g}(\omega - \omega_0)$$

Absorption rate

$$\Gamma_\mathrm{g}(\omega - \omega_0) = -\frac{1}{2}\alpha''(\omega - \omega_0)\left\langle E^2 \right\rangle_t$$

Recoil limit

$$kT_r = \frac{\hbar^2 q^2}{2m} \quad \Rightarrow \quad T_r \approx 1\,\mu\mathrm{K}$$

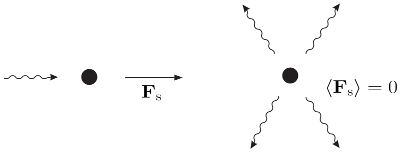


Figure : Absorption of a photon followed by spontaneous emission in a random direction.

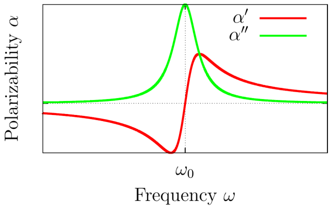


Figure : Real (red) and imaginary part (green) of the polarizability over frequency.

Zeeman slower

Doppler effect

$$\omega = \omega_0 - kv$$

Magnetic field cancels decreasing Doppler shift

$$\begin{aligned} \hbar\omega &= \hbar\omega_0 - V_D + V_z \\ &= \hbar\omega_0 - \hbar kv + gm\mu_B B \end{aligned}$$

Constant deceleration of atoms on resonance

$$-\dot{v} = a = \frac{F_s}{m} \approx 10^5 \frac{m}{s^2}$$

$$v_0^2 - v^2 = 2az \Rightarrow v(z) = v_0\sqrt{1 - \frac{2a}{v_0^2} z}$$

Figure : An atom moving towards a laser beam

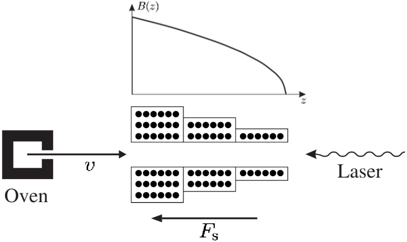


Figure : Zeeman slower. From: Foot - Atomic Physics

Doppler cooling

Doppler effect

$$\omega = \omega_0 - kv$$

$\Rightarrow$ Use red-detuned lasers ($\omega < \omega_0$)

Force of two counter-propagating laser beams

$$\begin{aligned} F &= F_s(\omega - \omega_0 + kv) \\ &\quad - F_s(\omega - \omega_0 - kv) \\ &\approx -\beta v \end{aligned}$$

Atoms are cooled, but not trapped.

$\Rightarrow$ MOT: Add a spacial dependence

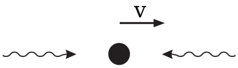


Figure : Moving atom in two counter-propagating laser beams.

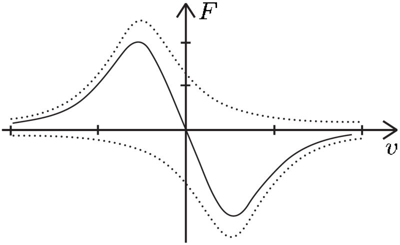


Figure : Force over the velocity of an atom. From: Foot - Atomic Physics

Magneto-optical trap

Magnetic quadrupole field: Linear Zeeman shift at origin

$$V_Z = g m \mu_B B \propto r$$

Counter-propagating laser beams: Red-detuned, σ_- and σ_+ polarized

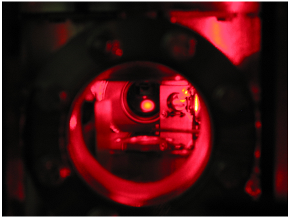


Figure : Trapped lithium atoms. From: Quantum optics group at ETH Zürich

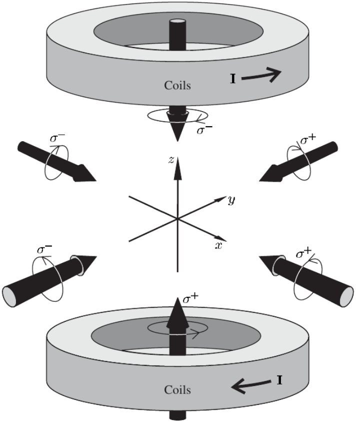
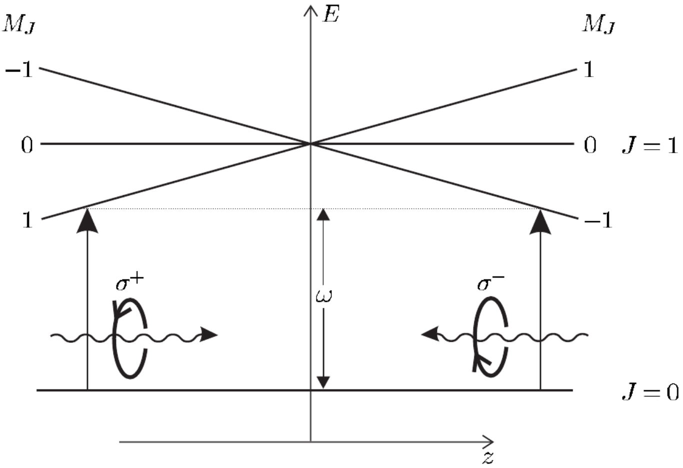


Figure : Setup of a MOT. From: Foot - Atomic Physics

Magneto-optical trap



Transitions
σ^-: $\Delta m = -1$
σ^+: $\Delta m = +1$

Figure : Schematic of a 1D MOT with a $J = 0 \leftrightarrow J = 1$ transition. From: Foot - Atomic Physics

Magnetic trap

Harmonic trap

$$B = \frac{B''}{2} r^2 + B_0$$

Zeeman effect

$$V_Z(\mathbf{r}) = gm\mu_B B$$

Trapping force

$$\mathbf{F}(\mathbf{r}) = -\nabla V_Z \propto -gm\,\mathbf{r}$$

Low-field seekers ($gm > 0$) are trapped, high-field seekers ($gm < 0$) are repelled.

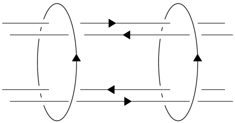


Figure : Currents in an Ioffe-Pritchard trap inducing a harmonic magnetic field. From: Pethick, Smith - BEC in dilute gases

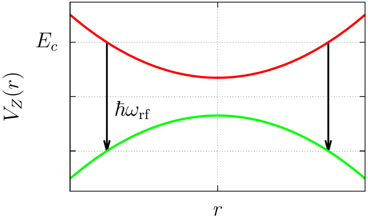


Figure : Magnetic trap potential with a low-field seeking state (red) and a high-field seeking state (green).

Evaporative cooling

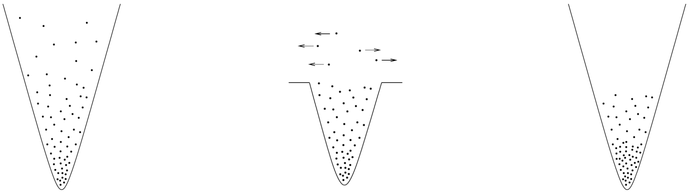


Figure : Atoms in a harmonic potential. From: Pethick, Smith - BEC in dilute gases

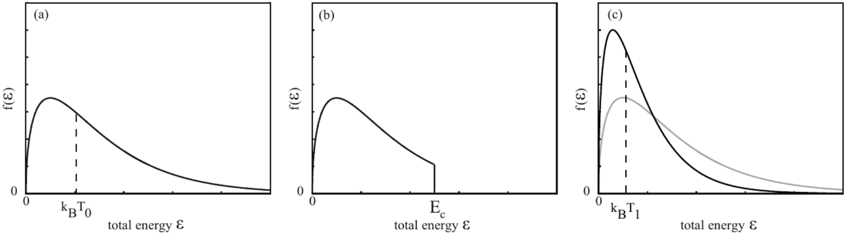


Figure : (a) Maxwell-Boltzmann energy distribution at $T = T_0$, (b) Cut-off of "hot" atoms with $E > E_c$, (c) New equilibrium temperature $T_1 < T_0$. From: Diploma thesis at PI5.

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Thomas-Fermi approximation

Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V_{\mathrm{ho}}(\mathbf{r}) + g|\Psi|^2\right]\Psi = \mu\Psi$$

Coupling constant $g = \frac{4\pi\hbar^2}{m}a > 0$ for repulsive interactions.

Thomas-Fermi approximation

For $\frac{Na}{a_{\mathrm{ho}}} \gg 1$ kinetic energy is small and can be neglected:

$$\left[V_{\mathrm{ho}}(\mathbf{r}) + g|\Psi|^2\right]\Psi = \mu\Psi$$

Blackboard:
Density distribution, chemical potential and cloud radius.

Thomas-Fermi approximation

Thomas-Fermi approximation

Density distribution for $r \leq r_{\mathrm{TF}}$

$$n(\mathbf{r}) = \frac{\mu}{g}\left(1 - \frac{r^2}{r_{\mathrm{TF}}^2}\right)$$

Chemical potential

$$\mu = \frac{1}{2}\hbar\omega_{\mathrm{ho}}\left(15\,\frac{Na}{a_{\mathrm{ho}}}\right)^{2/5}$$

Spatial extent of cloud

$$r_{\mathrm{TF}} = a_{\mathrm{ho}}\left(15\,\frac{Na}{a_{\mathrm{ho}}}\right)^{1/5} > a_{\mathrm{ho}}$$

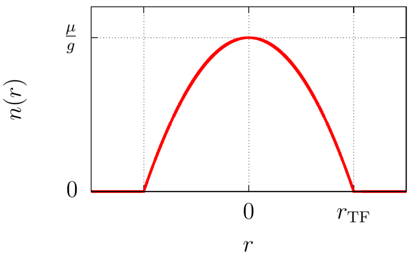

Figure : Density distribution in a harmonic trap.

Thomas-Fermi approximation

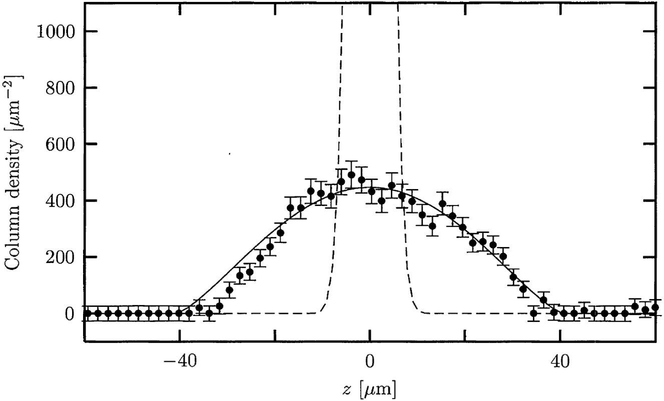

Figure : Density distribution of sodium atoms. From: Dalfovo et al. (1999)

Healing length

Kinetic energy = interaction energy

$$\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m\xi^2} = g\, n(\mathbf{r})$$

$$\Rightarrow \quad \xi^2 = \frac{\hbar^2}{2mgn} = \frac{1}{8\pi a n}$$

Solving the GPE for a box potential

$$\Psi(r) = \Psi_0 \tanh\left(\frac{r}{\sqrt{2}\xi}\right)$$

Length scale on which perturbations in the density distribution are "healed".

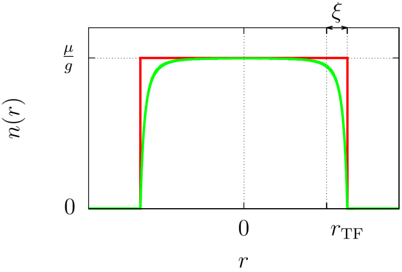

Figure : Density distribution in a box potential within Thomas-Fermi approximation (red) and solution of the GPE (green)

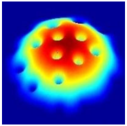

Figure : Vortices in a BEC. From: Australian Centre for Quantum-Atom Optics

Conclusion

- Ideal bose gas
  - Critical temperature
  - Condensate fraction
- Experimental techniques
  - Laser cooling
  - Evaporative cooling
- Weakly interacting bose gas
  - Parabola shaped density distribution
  - Spatial extent of the cloud
  - Healing length

Thank you for your attention.