Bose-Einstein condensation of cold gases in traps

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1 Ideal bose gas

1.1 Harmonic potential

Isotropic harmonic potential and oscillator length

$$V_{\text{ho}}(\mathbf{r}) = \frac{1}{2} m\omega_{\text{ho}}^2 \mathbf{r}^2 \qquad\qquad a_{\text{ho}} = \sqrt{\frac{\hbar}{m\omega_{\text{ho}}}}$$

1.2 Critical temperature T_c

Bose distribution and density of states

$$f(E) = \frac{1}{e^{(E-\mu)/kT} - 1} \qquad\qquad g(E) = \frac{1}{2(\hbar\omega_{\text{ho}})^3} E^2$$

Total number of particles in excited states at T_c

$$N = N_{\text{ex}}(T = T_c, \mu = 0) = \int_0^\infty \mathrm{d}E \, \frac{g(E)}{e^{E/kT_c} - 1} = \zeta(3) \left(\frac{kT_c}{\hbar\omega_{\text{ho}}} \right)^3$$

Critical temperature

$$kT_c \approx 0.94 \, \hbar\omega_{\text{ho}} N^{1/3} \qquad \Rightarrow \qquad T_c \approx 500 \, \text{nK}$$

2 Experimental techniques

2.1 Radiation forces

Scattering force: Absorption of a photon $\hbar\mathbf{q}$ followed by spontaneous emission in a random direction

$$\mathbf{F}_{\text{scatt}} = \dot{\mathbf{p}} = \hbar\mathbf{q} \, \Gamma_{\text{g}}(\omega)$$

All optical cooling techniques are limited by the recoil of an emitted a photon

$$kT_r = \frac{\hbar^2 q^2}{2m} \quad \Rightarrow \quad T_r \approx 1 \, \mu\text{K}$$

Dipole force: Stark shift of the time-averaged electric field of the laser leads to a force towards higher laser intensities.

2.2 Zeeman slower

Zeeman shift V_Z cancels decreasing Doppler shift V_D while atoms are being slowed down

$$\hbar\omega = \hbar\omega_0 + V_Z + V_D = \hbar\omega_0 + gm\mu_B B - kv$$

2.3 Magneto-optical trap

A linear magnetic field at the center of the trap and counter-propagating σ^+ and σ^- polarized laser beams lead to a trapping and cooling force towards the origin of the trap.

2.4 Magnetic trap and evaporative cooling

Low-field seekers ($gm > 0$) are trapped within the harmonic trap potential

$$B = \frac{B''}{2} r^2 + B_0 \qquad \text{and} \qquad V_Z(\mathbf{r}) = gm\mu_B B \qquad \Rightarrow \qquad \mathbf{F}(\mathbf{r}) = -\nabla V_Z \propto -gm\,\mathbf{r}$$

Loss of atoms above a cut-off energy leads to a lower temperature of the ensemble.

3 Weakly interacting bose gas

3.1 Thomas-Fermi approximation

Neglect kinetic energy in the Gross-Pitaevskii equation for $\frac{Na}{a_{\rm ho}} \gg 1$

$$\left[V_{\rm ho}(\mathbf{r}) + g|\Psi|^2\right] \Psi = \mu\Psi$$

Density distribution for $r \leq r_{\rm TF}$

$$n(\mathbf{r}) = \frac{\mu}{g}\left(1 - \frac{r^2}{r_{\rm TF}^2}\right) \qquad \text{with} \qquad r_{\rm TF} = a_{\rm ho}\left(15\,\frac{Na}{a_{\rm ho}}\right)^{1/5} > a_{\rm ho}$$

3.2 Healing length

Kinetic energy comparable to interaction energy

$$\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m\xi^2} = g\,n \qquad \Rightarrow \qquad \xi^2 = \frac{\hbar^2}{2mgn} = \frac{1}{8\pi an}$$

ξ is the length scale on which perturbations in the density distribution are “healed”.