Rotational Properties of Bose - Einstein Condensates

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Outline

Motivation

- one of the hallmarks of superfluids is their response to rotation
- special properties are consequence of their constrained motions
- circulation around a closed contour is quantized

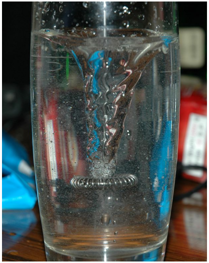
Classical gases

circulation along a closed line

$$\Gamma = \oint v dl$$

rewriting with Stokes theorem

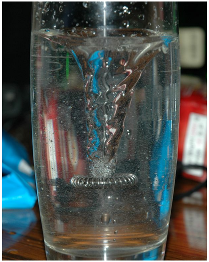
$$\oint v dl = \int (\nabla \times v) dA$$

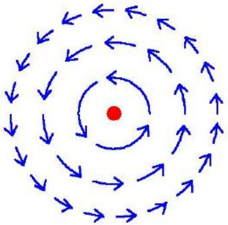


http://de.wikipedia.org/wiki/Wirbel_%28Str%C3%B6mungslehre%29 velocity of a circulation

$$\nabla \times v = \nabla \times (\Omega \times r) \neq 0$$

circulation in a classical gas can take arbitrary values
$\rightarrow$ difference to BEC





http://london.ucdavis.edu/Research/helium_sf.html
http://de.wikipedia.org/wiki/Wirbel_%28Str%C3%B6mungslehre%29

Circulation in BEC

velocity:

$$v = \frac{\hbar}{m}\nabla\phi$$

$$\nabla \times v = 0$$

field of velocity is irrotational unless there is a singularity in the phase of the wave function
$\rightarrow$ possible motions are restricted

change of the wave function:

$$\Delta\phi = \oint \nabla\phi dl = 2\pi l$$

$$\Gamma = \oint v dl = \frac{2\pi\hbar}{m} \cdot l$$

circulation is quantized

assume a trap invariant about rotation around z axis with distance ρ from the axis

$$v_\phi = l \frac{\hbar}{m\rho}$$

- countour encloses the axis the circulation has to be $\frac{2\pi l \hbar}{m}$, otherwise zero
- if $l \neq 0$ the wave function has to vanish on the axis of the trap

- external potential with axial symmetry, state has a singularity on the axis
- each particle carries momentum $l\hbar$
- total angular momentum is $Nl\hbar$ with the particle number N if the singularity lies off the axis the angular momentum generally differs from this value

generalization of $\nabla \times v = 0$:

$$\nabla \times v = \frac{2\pi l \hbar}{m} \delta^2(\rho) e_z$$

case of many vortices

$$\nabla \times v = \sum \frac{2\pi l \hbar}{m} \delta^2(\rho) e_z$$

Single vortex

Time independent Gross-Pitaevskii equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r) + V(r)\psi(r) + U_0|\psi(r)|^2\psi(r) = \mu\psi(r)$$

with the energy E of the system

$$E(\psi) = \int dr \left[\frac{\hbar^2}{2m}|\nabla\psi(r)|^2 + V(r)|\psi(r)|^2 + \frac{1}{2}U_0|\psi(r)|^4\right]$$

consider a trap with an axial symmetry, then the condensate wave function can be written in cylindrical coordinates

$$\psi(r) = f(\rho, z)e^{il\phi}$$

for the energy you get

$$E = \int dr \left[\frac{\hbar^2}{2m} \left[\left(\frac{\partial f}{\partial \rho} \right)^2 + \left(\frac{\partial f}{\partial z} \right)^2 \right] + \frac{\hbar^2}{2m} l^2 \frac{f^2}{\rho^2} + V(\rho, z) f^2 + \frac{U_0}{2} f^4 \right]$$

if you insert the wave function in the GP-equation you get a equation for the amplitude of the wave function

$$-\frac{\hbar^2}{2m} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{\partial^2 f}{\partial z^2} \right] + \frac{\hbar^2}{2m\rho^2} l^2 f + V(\rho, z) f + U_0 f^3 = \mu f$$

single vortex in a uniform medium

- consider an infite medium with a uniform potential which can be taken to be zero: $V(\rho, z) = 0$
- wave function does not depend on z in the ground state, hence terms with a derivative with respect to z vanish
- only consider the case for l=1 because of the physical importance of single quantized vortices

- large distances from the axis the radial term and the barrier term ($\propto \frac{1}{\rho^2}$) become unimportant

  $\rightarrow$ magnitude of the wave function: $f(\rho \rightarrow \infty) = f_0 = \left(\frac{\mu}{U_0}\right)^{\frac{1}{2}}$
- close to the axis the derivative and the centrifugal term dominate
- the crossover between the two different behaviours occurs at distances of the order of the healing length ξ, hence the length can scaled in units of the healing length ξ:

$$\frac{\hbar^2}{2m\xi^2} = nU_0 = \mu$$

with the density $n = f_0^2$ at large distances introduce the new variable

$$x = \frac{\rho}{\xi}$$

scale the amplitude of the wave function to its value far from the axis

$$\kappa = \frac{f}{f_0}$$

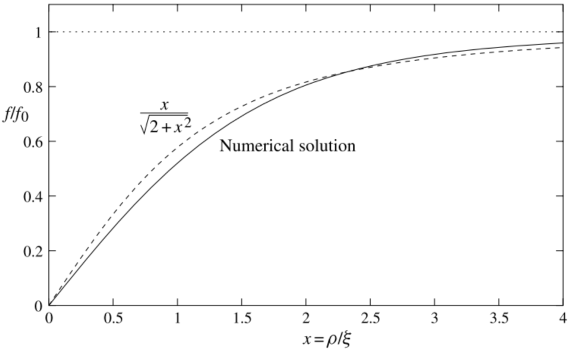
then you can rewrite the energy

$$E = n^2 U_0 \left[\left(\frac{d\kappa}{dx} \right)^2 + \frac{\kappa^2}{x^2} + \frac{1}{2}\kappa^4 \right]$$

the Gross-Pitaevskii equation becomes:

$$-\frac{1}{x}\frac{d}{dx}\left(x\frac{d\kappa}{dx}\right)+\frac{\kappa}{x^2}+\kappa^3-\kappa=0$$

this equation can be solved numerically



Bose-Einstein Condensation (Pethick, Smith)

Energy ϵ per unit length along the axis of the vortex

$$\epsilon = \int_0^{\rho_0} 2\pi\rho d\rho \left[\frac{\hbar^2}{2m} \left(\frac{df}{d\rho} \right)^2 + \frac{\hbar^2}{2m} \frac{f^2}{\rho^2} + \frac{U_0}{2} f^4 \right]$$

to get well defined results only consider the energy of atoms in a finite distance ρ_0 of the vortex with $\rho_0 >> \xi$
energy of a vortex:

$$\epsilon_v = \epsilon - \epsilon_0$$

- energy of a uniform gas per unit volume is given by $\bar{n}^2 U_0/2$ with the average density $\bar{n} = \nu/\pi D^2$
- the density far from the axis in the vortex state is not equal to the average density of the uniform system, because the vortex state has a hole in the density distribution
- both states have the same number of particles so the density of the vortex state far from the axis is larger than the density in the uniform state

Number of particles per unit length

$$\nu = \int_0^D d\rho\ 2\pi\rho f^2 = \pi D^2 f_0^2 - \int_0^D 2\pi\rho d\rho (f_0^2 - f^2)$$

Energy per unit length of the uniform system

$$\epsilon_0 = \frac{1}{2}\pi D^2 f_0^4 U_0 - f_0^2 U_0 \int_0^D 2\pi\rho d\rho (f_0^2 - f^2)$$

the energy ϵ_v per unit length associated with a vortex:

$$\epsilon_v = \epsilon - \epsilon_0$$

$$\epsilon_v = \int_0^D 2\pi\rho d\rho \left[\frac{\hbar^2}{2m}\left(\frac{df}{d\rho}\right)^2 + \frac{\hbar^2}{2m}\frac{f^2}{\rho^2} + \frac{U_0}{2}(f_0^2 - f^2)^2\right]$$

$$= \frac{\pi\hbar^2}{m} n \int_0^{D/\xi} x dx \left[\left(\frac{d\kappa}{dx}\right) + \frac{\kappa^2}{x^2} + \frac{1}{2}(1-\kappa^2)^2\right]$$

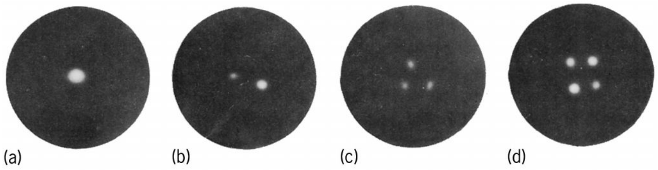
Evaluating the result for the numerical solution of the Gross-Pitaevskii equation gives you

$$\epsilon_v = \pi n \frac{\hbar^2}{m} \ln\left(1.464 \frac{D}{\xi}\right)$$

Vortex lattice

if the rotation rate is increased the nature of the equilibrium state is changed

- two vortices are rotating around each other
- three vortices are ordered in a triangle
- more vortices are ordered in a regular array

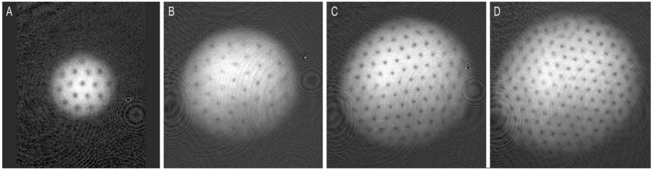


http://encyclopedia2.thefreedictionary.com/Quantized+vortices

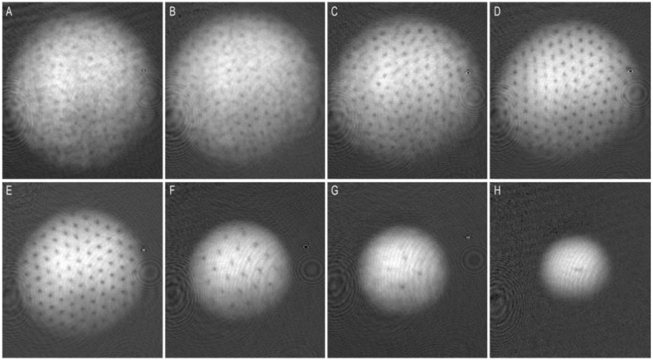
Vortex latttice

- the group of Ketterle described in their paper (Science, (2001)) the observation of vortex lattices in rotating Bose-Einstein Condensates
- they used a laser to rotate the condensate
- they varied:
  $\rightarrow$ stirring time
  $\rightarrow$ time to equlibriate

- after producing the condensate they switched off the trap after a variable stirring time and did absorption pictures
- they observed highly ordered vortex lattices with up to 130 vortices



Observation of Vortex Lattices in Bose-Einstein Condensates, Ketterle the vortices have a lifetime of several seconds up to 40 s



Observation of Vortex Lattices in Bose-Einstein Condensates, Ketterle

the vortices formed a regulary lattice after 500 ms

Summary

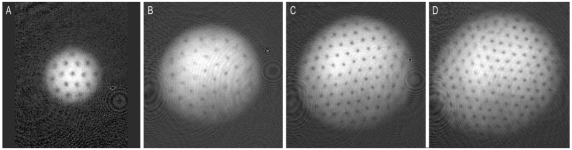
- velocity in a BEC:

$$v = \frac{\hbar}{m}\nabla\phi$$

- energy of a vortex in uniform medium

$$\epsilon_v = \int_0^D 2\pi\rho d\rho \left[\frac{\hbar^2}{2m}\left(\frac{df}{d\rho}\right)^2 + \frac{\hbar^2}{2m}\frac{f^2}{\rho^2} + \frac{U_0}{2}(f_0^2 - f^2)^2\right]$$

- at higher rotation rate the number of vortices is increased



Thank you for your attention