Rotational Properties of Bose-Einstein Condensates

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Seminar Cold Gases

1 Classical Gases

Circulation along a closed line:

$$\Gamma = \oint v dl$$

Can be rewritten with the stokes theorem

$$\oint v dl = \int (\nabla \times v) dA$$

In a classical gas, the velocity is given by:

$$\nabla \times v = \nabla \times (\Omega \times r) \neq 0$$

The circulation in a classical gas can take arbitrary values

2 Rotation of BEC

In a BEC the velocity and the rotation of the velocity are given by:

$$v = \frac{\hbar}{m} \nabla \phi$$

$$\nabla \times v = 0$$

Thus the velocity is irrotational unless there is a singularity in the phase of the wavefunction, hence the possible motions are restricted
The circualtion is quantized with the quantum number l:

$$\Gamma = \oint v dl = \frac{2\pi\hbar}{m} \cdot l$$

As a generalization of the above equation you get:

$$\nabla \times v = \frac{2\pi\hbar l}{m} \delta^2(\rho) \hat{e}_z$$

In the case of many vortices the right side of the equation becomes a sum

2.1 Single vortex in an uniform medium

Assuming the wave function $\psi(r) = f(\rho, z) e^{il\phi}$ and inserting this function into the Gross-Pitaevskii equation you get formula for the energy and for the amplitude of the wave function:

$$E = \int dr \left[\frac{\hbar^2}{2m} \left[\left(\frac{\partial f}{\partial \rho} \right)^2 + \left(\frac{\partial f}{\partial z} \right)^2 \right] + \frac{\hbar^2}{2m} l^2 \frac{f^2}{\rho^2} + V(\rho, z) f^2 + \frac{U_0}{2} f^4 \right]$$

$$-\frac{\hbar^2}{2m} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{\partial^2 f}{\partial z^2} \right] + \frac{\hbar^2}{2m\rho^2} l^2 f + V(\rho, z) f + U_0 f^3 = \mu f$$

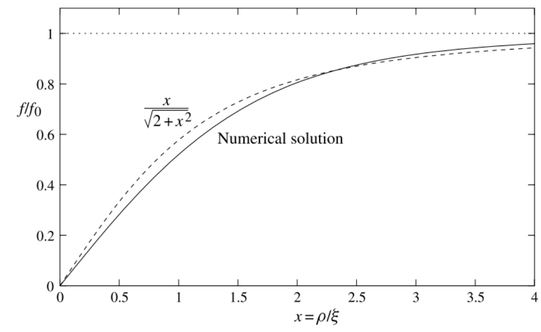
If you consider a infinite medium with uniform potential $V(\rho, z) = 0$ and put $l = 1$ then you can rewrite the equations after introducing some new variables with the healing length ξ and the amplitude f_0 far from the axis:

$$x = \frac{\rho}{\xi}$$

$$\kappa = \frac{f}{f_0}$$

$$E = n^2 U_0 \left[\left(\frac{d\kappa}{dx} \right)^2 + \frac{\kappa^2}{x^2} + \frac{1}{2}\kappa^4 \right]$$

$$-\frac{1}{x}\frac{d}{dx}\left(x\frac{d\kappa}{dx} \right) + \frac{\kappa}{x^2} + \kappa^3 - \kappa = 0$$



Energy ϵ_v associated with a vortex:

$$\epsilon_v = \epsilon - \epsilon_0$$

$$\epsilon_v = \int_0^D 2\pi\rho d\rho \left[\frac{\hbar^2}{2m} \left(\frac{df}{d\rho} \right)^2 + \frac{\hbar^2}{2m}\frac{f^2}{\rho^2} + \frac{U_0}{2}(f_0^2 - f^2)^2 \right]$$
$$= \frac{\pi\hbar^2}{m} n \int_0^{D/\xi} x dx \left[\left(\frac{d\kappa}{dx} \right) + \frac{\kappa^2}{x^2} + \frac{1}{2}(1 - \kappa^2)^2 \right]$$

3 Vortex lattice

At higher rotation rates the equlibirum state is changing

The Ketterle group is describing in their paper vortex lattices in rotating Bose Einstein Condensates, they observed up to 130 vortices with different lifetimes up to 40 s