Hauptseminar: Physik der kalten Gase
Dipolar gases

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Motivation

• 1995: Bose-Einstein condensation in diluted gases → nobel prize 2001 (Cornell, Wieman, Ketterle)
• 2005: Cr-BEC
• better understanding of static and dynamical properties of weakly interacting Bose gases
• strongly correlated systems reached with ultra-cold atoms and/or molecules
• contact-interaction: short-range and isotropic vs. dipole-dipole interaction: long-range and anisotropic
• supersolids

Dipole-dipole interaction

• Normally the interaction potential between two atoms with distance $r$ is the Van-der-Waals potential ($\propto \frac{1}{r^6}$)
• For vanishing collision energy only the s-wave scattering plays an important role
• Ultra-cold regime: characterization of the potential by a pseudopotential with scattering length $a$ and the contact interaction strength $g$

$$U_{\text{contact}}(r) = \frac{4\pi \hbar^2 a}{m} \delta(r) \equiv g \delta(r)$$

• Dipole-dipole interaction can be written as:

$$U_{dd}(r) = \frac{C_{dd}}{4\pi} \left( e_1 \cdot e_2 \right) \frac{r^2}{r^6} - \frac{3}{r^3} \left( e_1 r \right) \left( e_2 r \right)$$

$$U_{dd}(r, r') = \frac{C_{dd}}{4\pi} \frac{1 - 3 \cos^2 \theta}{r^3}$$

$$C_{dd} = \mu_0 \mu^2$$  
(permanent magnetic dipole moment)

$$C_{dd} = \frac{d^2}{\epsilon_0}$$  
(permanent electric dipole moment)

• Dipole-dipole interaction has two important properties anisotropy and long-range character
• strength of the dipole-dipole interaction and ratio to the s-wave scattering length:

$$a_{dd} = \frac{C_{dd} m}{12\pi \hbar^2}$$

$$\epsilon_{dd} = \frac{a_{dd}}{a} = \frac{C_{dd}}{3g}$$

• Feshbach resonances for tuning the scattering length $a$
Gross-Pitaevskii equation (GPE)  
- time dependent GPE  
\[ i\hbar \frac{\partial}{\partial t} \psi(r, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + U_{rr}(r) + \frac{g}{2} |\psi(r, t)|^2 + \frac{C_{dd}}{8\pi} \int d\mathbf{r}' \frac{1 - 3 \cos^2 \theta}{|\mathbf{r} - \mathbf{r}'|^3} |\psi(\mathbf{r}', t)|^2 \right] \psi(r) \]

Several numerical and approximate methods for the solution of the stationary GPE.

1. Gaussian ansatz
2. Thomas-Fermi approximation (neglection of the kinetic energy)

Instabilities
- Partially attractive character of the dipole-dipole interaction leads to a stability problem of the dipolar BEC excitation
- Spectrum of a homogeneous dipolar condensate:
\[ \omega = k \sqrt{\frac{n_0}{m}} \left[ g + \frac{C_{dd}}{3} (3 \cos^2 \alpha - 1) \right] + \frac{\hbar^2 k^2}{4m} \]

together with the definition
\[ \epsilon_{dd} = \frac{a_{dd}}{a} = \frac{C_{dd}}{3g} \]

- Metastable for \( \epsilon_{dd} > 1 \)
- Most unstable situation for \( \alpha = \pi/2 \)

Excitations of the system
- Instabilities in dipolar gases due to excitations in the system (even in the pancake traps)
- Gas feels the 3D nature of the dipolar interactions (e.g. their partially attractive character) → 'roton-maxon' instability

Summary and outlook
- New effects by studying the dipole-dipole interaction due to anisotropy and long-range character
- Gross-Pitaevskii equation for calculations, Gaussian ansatz and Thomas-Fermi approximation
- Instabilities, form of the condensate, minimization of the energy to get \( a_{\text{crit}}(\lambda) \), roton-maxon instability
- Beside BEC, atomic degenerate Fermi gases (Fermi superfluidity in the weak interaction limit by BCS theory)
- BEC-BCS crossover in the limit of strong correlations