Hauptseminar: Physik der kalten Gase
Dipolar gases

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Motivation

- 1995: Bose-Einstein condensation in diluted gases $\rightarrow$ nobel prize 2001 (Cornell, Wieman, Ketterle)
- 2005: Cr-BEC
- better understanding of static and dynamical properties of weakly interacting Bose gases
- strongly correlated systems reached with ultra-cold atoms and/or molecules
- contact-interaction: short-range and isotropic vs. dipole-dipole interaction: long-range and anisotropic
- supersolids

Dipole-dipole interaction

- Normally the interaction potential between two atoms with distance r is the Van-der-Waals potential ($\propto \frac{1}{r^6}$)
- For vanishing collision energy only the s-wave scattering plays an important role
- Ultra-cold regime: characterization of the potential by a pseudopotential with scattering length a and the contact interaction strength g

$$U_{\text{contact}}(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}) \equiv g\delta(\mathbf{r})$$

- Dipole-dipole interaction can be written as:

$$U_{\text{dd}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi}\frac{(e_1 \cdot e_2)\, r^2 - 3\,(e_1\mathbf{r})\,(e_2\mathbf{r})}{r^5}$$

$$U_{\text{dd}}(\mathbf{r},\, \mathbf{r}') = \frac{C_{\text{dd}}}{4\pi}\frac{1 - 3\cos^2\theta}{r^3}$$

$$\begin{aligned} C_{\text{dd}} &= \mu_0\mu^2 \\ &\quad \text{(permanent magnetic dipole moment)} \\ C_{\text{dd}} &= \frac{d^2}{\epsilon_0} \\ &\quad \text{(permanent electric dipole moment)} \end{aligned}$$

- Dipole-dipole interaction has two important properties anisotropy and long-range character
- strength of the dipole-dipole interaction and ratio to the s-wave scattering length:

$$a_{\text{dd}} \equiv \frac{C_{\text{dd}} m}{12\pi\hbar^2}$$

$$\epsilon_{\text{dd}} \equiv \frac{a_{\text{dd}}}{a} = \frac{C_{\text{dd}}}{3g}$$

- Feshbach resonances for tuning the scattering length a

Gross-Pitaevskii equation (GPE)

- time dependent GPE

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},\,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + U_{\mathrm{tr}}(\mathbf{r}) + \frac{g}{2}\left|\psi(\mathbf{r},\,t)\right|^2 + \frac{C_{dd}}{8\pi}\int d\mathbf{r}'\frac{1-3\cos^2\theta}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\left|\psi(\mathbf{r}',\,t)\right|^2\right]\psi(\mathbf{r})$$

Several numerical and approximate methods for the solution of the stationary GPE.

1. Gaussian ansatz
2. Thomas-Fermi approximation (neglection of the kinetic energy)

Instabilities

- Partially attractive character of the dipole-dipole interaction leads to a stability problem of the dipolar BEC excitation
- spectrum of a homogeneous dipolar condensate:

$$\omega = k\sqrt{\frac{n_0}{m}\left[g + \frac{C_{\mathrm{dd}}}{3}\left(3\cos^2\alpha - 1\right)\right] + \frac{\hbar^2 k^2}{4m^2}}$$

together with the definition

$$\epsilon_{\mathrm{dd}} \equiv \frac{a_{\mathrm{dd}}}{a} = \frac{C_{\mathrm{dd}}}{3g}$$

- metastable for $\epsilon_{\mathrm{dd}} > 1$
- most unstable situation for $\alpha = \pi/2$

excitations of the system

- Instabilities in dipolar gases due to excitations in the system (even in the pancake traps)
- gas feels the 3D nature of the dipolar interactions (e.g. their partially attracticve character) → 'roton-maxon' instability

Summary and outlook

- New effects by studying the dipole-dipole interaction due to anisotropy and long-range character
- Gross-Pitaevskii equation for calculations, Gaussian ansatz and Thomas-Fermi approximation
- Instabilities, form of the condensate, minimization of the energy to get $a_{\mathrm{crit}}(\lambda)$, roton-maxon instability
- Beside BEC, atomic degenerate Fermi gases (Fermi superfluidity in the weak interaction limit by BCS theory)
- BEC-BCS crossover in the limit of strong correlations