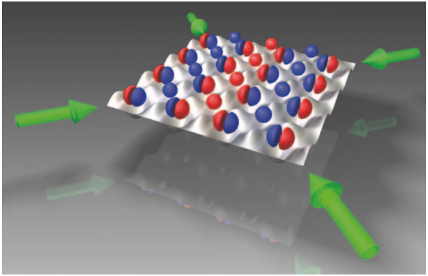
Tilted Bose Hubbard model

Phase transition in 1D



Physik Journal 12 (2013) Nr.1 p.27

Bruno Villa
Institut für theoretische Physik III

Supervisor:
Alexander Krupp

27/05/2013

Outline

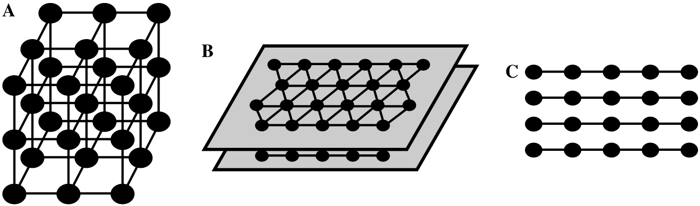
Motivation

- Set of problems with no viable classical computing simulation
- Controllable environment for quantum simulations
  - Quantum magnetism
  - High temperature superconductivity
  - Quantum computing

Motivation

Optical lattices

- Spatially periodic structure of electric fields
- Usually generated by superposition of laser beams



Jaksch, D. et al. (2005) *Annals of Physics* **315(1)**, 52–79

Fundamentals

The Tilted Bose Hubbard Model (BHM)

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) + \frac{U}{2} \sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j$$

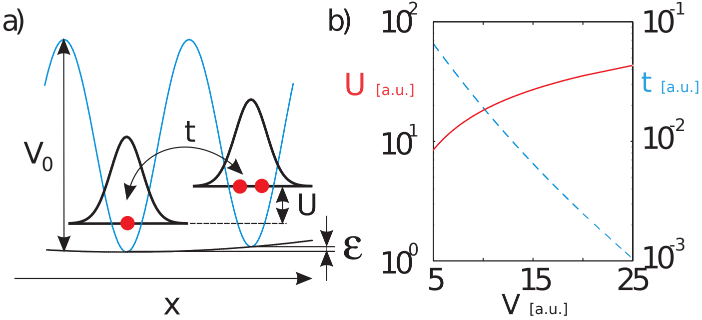
Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)

$$\mathcal{H} = -t\sum_{\langle i,j\rangle}(\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) + \frac{U}{2}\sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j$$
$$- E\sum_j \mathbf{e}\cdot\mathbf{r}_j \hat{b}_j^\dagger \hat{b}_j$$

Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)

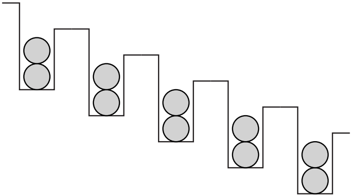


Jaksch, D. et al. (1998) *Physical Review Letters* pp. 5–8

The Tilted Bose Hubbard Model (BHM)

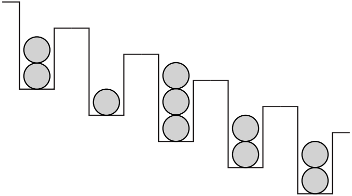
Mott insulator

- Minimize $\mathcal{H}$ in limit $U \gg t$
- Average site occupation n_0



Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

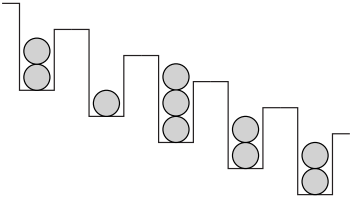
- Tunneling generates new state
- Energy difference $U - E$



Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

Define dipole operator

$$\hat{d}_j^\dagger = \frac{\hat{b}_j \hat{b}_{j+1}^\dagger}{\sqrt{n_0(n_0+1)}}$$



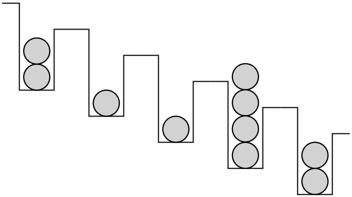
with the conditions

$$\hat{d}_j^\dagger \hat{d}_j \leq 1$$

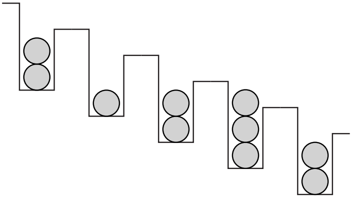
$$\hat{d}_j^\dagger \hat{d}_j \hat{d}_{j+1}^\dagger \hat{d}_{j+1} = 0$$

Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)



$\mathcal{O}(t^2)$



Dipole of length 2

Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)

Rewrite Hamiltonian

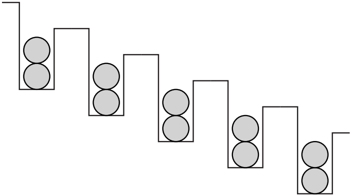
$$\mathcal{H} = -t\sum_{\langle i,j\rangle}(\hat{b}_i^\dagger\hat{b}_j + \hat{b}_j^\dagger\hat{b}_i) + \frac{U}{2}\sum_j \hat{b}_j^\dagger\hat{b}_j^\dagger\hat{b}_j\hat{b}_j - E\sum_j \mathbf{e}\cdot\mathbf{r}_j\hat{b}_j^\dagger\hat{b}_j$$

with dipole operators

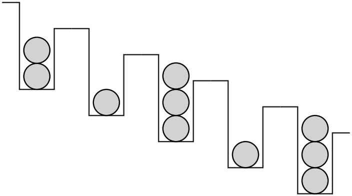
$$\mathcal{H}_d = -t\sqrt{n_0(n_0+1)}\sum_j(\hat{d}_j + \hat{d}_j^\dagger) + (U-E)\sum_j \hat{d}_j^\dagger\hat{d}_j$$

The Tilted Bose Hubbard Model (BHM)

Mott Insulator (MI)



Broken symmetry phase (BSP)



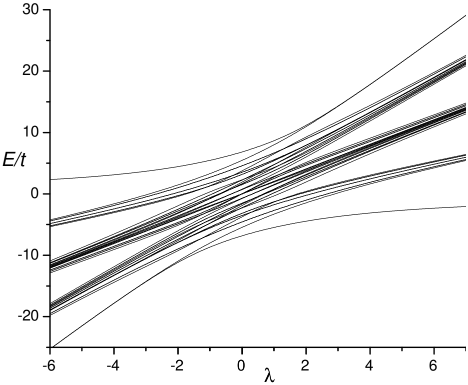
Tilt increase

⟶

Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)

Eigenenergies of $\mathcal{H}_d$ dependend on n_0 and $\lambda = \frac{U-E}{t}$

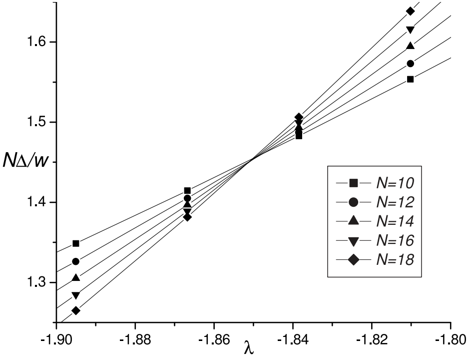


$N = 8,\ n_0 = 1$

Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

The Tilted Bose Hubbard Model (BHM)

Scaling $\Delta_E \propto N^{-z} = N^{-1}$ at λ_c



Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

Ising model mapping

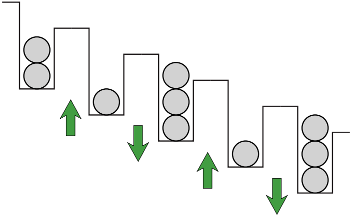
No tunneling event spin $\uparrow$

Tunneling event spin $\downarrow$

$$S_z^j = \frac{1}{2} - \hat{d}_j^\dagger \hat{d}_j$$

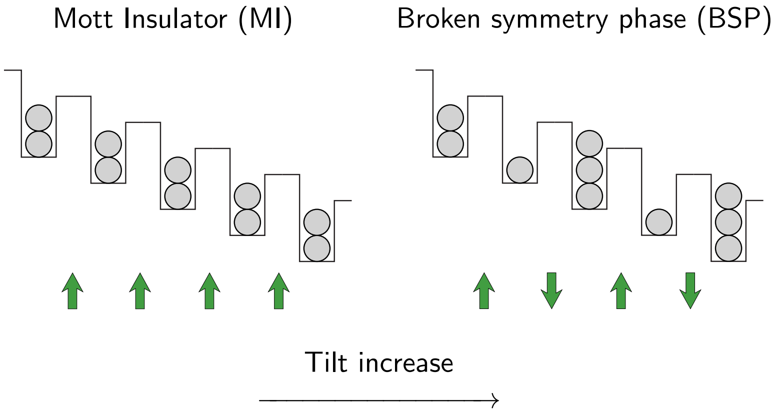
$$S_x^j = \frac{1}{2}\left(\hat{d}_j^\dagger + \hat{d}_j\right)$$

$$S_y^j = \frac{i}{2}\left(\hat{d}_j^\dagger - \hat{d}_j\right)$$



Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

Ising model mapping



Sachdev, S. et al. (2002) *Physical Review B* **8120**, 1–17

Ising model mapping
Constrains

- $\hat{d}_j^\dagger \hat{d}_j \leq 1$
  Fulfilled by definition

- $\hat{d}_j^\dagger \hat{d}_j \hat{d}_{j+1}^\dagger \hat{d}_{j+1} = 0$
  Add term to $\mathcal{H}_d$

$$J \hat{d}_{j+1}^\dagger \hat{d}_{j+1} \hat{d}_j^\dagger \hat{d}_j = J \left(S_z^{j+1} - 1/2 \right) \left(S_z^j - 1/2 \right)$$

Ising model mapping

- 1D Ising chain with longitudinal and transverse field equivalent to 2D Ising model

$$\mathcal{H}_s = J \sum_j (S_z^j S_z^{j+1} - h_x S_x^j - h_z S_z^j)$$

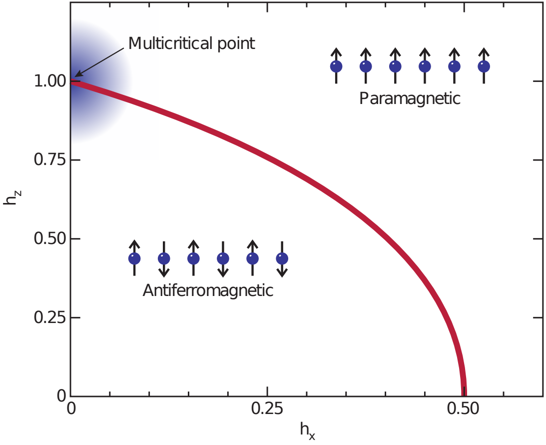
- No analytical solution

Ovchinnikov, A. A. (2003) *Physical Review B* **68.**

Ising model mapping

Phase transition

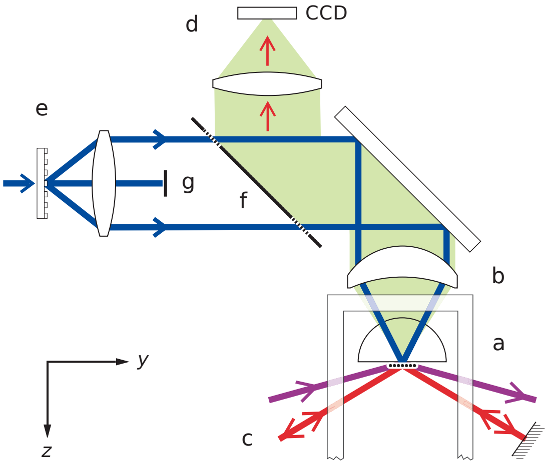
$$h_z = 1 - {}^{\Delta}\!/_{J}\,,\; h_x = 2\sqrt{2}\,{}^{t}\!/_{J}$$



Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

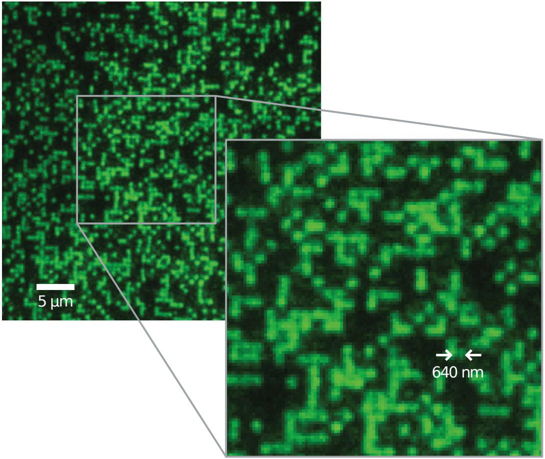
Experimental realization

Experimental realization
Setup



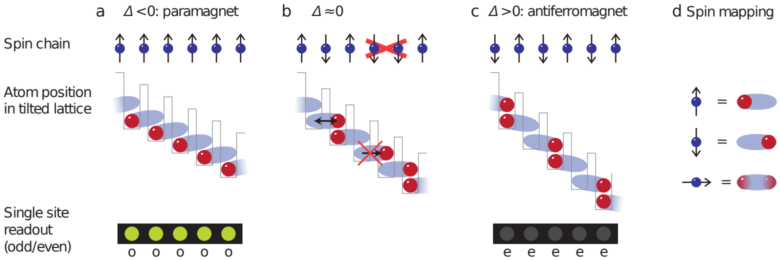
Bakr, Waseem S. et al. (2009) *Nature* **462(8482)**

Experimental realization
Setup



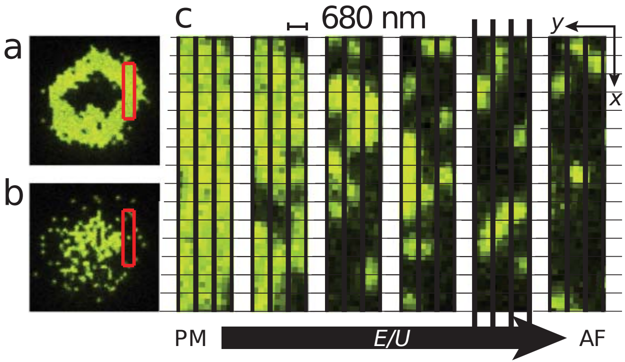
Bakr, Waseem S. et al. (2009) *Nature* **462(8482)**

Experimental realization
Setup



Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

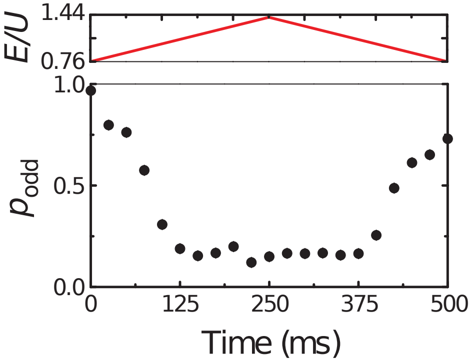
Experimental realization
Results



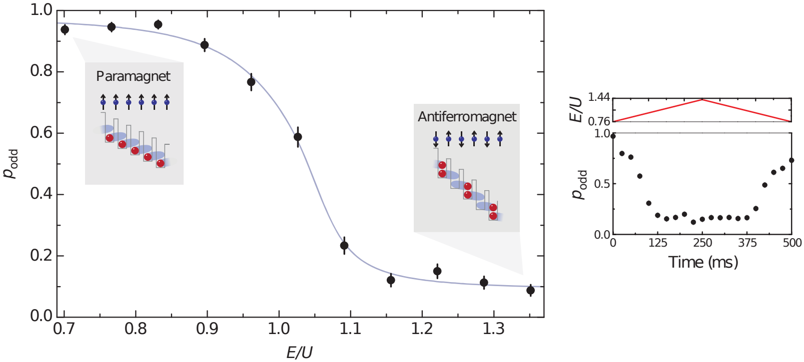
Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

Experimental realization
Results

$$\overline{S_z^j} = \frac{1}{2}\overline{p_{\text{odd}}^j}$$

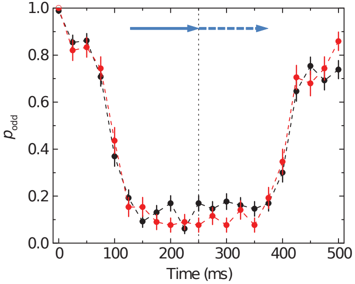


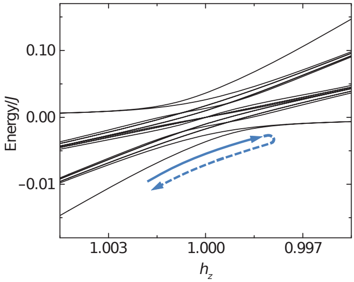
Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12



Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

Experimental realization
Results



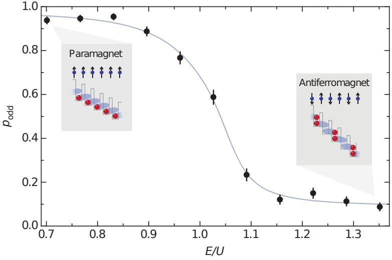


Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

Conclusions

Conclusions
Summary

- MI phase
- Tilted BHM
- Set of resonant states in 1D
- MI ground state — BSP
- Mapping to Ising model
- Experimental results



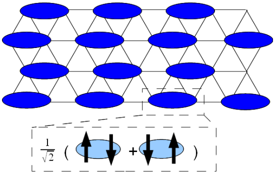
Simon, J. et al. (2011) *Nature* **472(7343)**, 307–12

Conclusions

Conclusions and outlook

- Experimental results in agreement with theory
- Optical lattices as a promising quantum simulator

- Higher dimensions
- Different lattice geometries



Wikimedia Commons — Valence bond solid

Thank you for your attention