Tilted Bose Hubbard model

Phase transition in 1D

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Outline

1. Motivation

2. Fundamentals
   - The Tilted Bose Hubbard Model (BHM)
   - Ising model mapping

3. Experimental realization
   - Setup
   - Results

4. Conclusions
Motivation

- Set of problems with no viable classical computing simulation
- Controllable environment for quantum simulations
  - Quantum magnetism
  - High temperature superconductivity
  - Quantum computing
Spatially periodic structure of electric fields

Usually generated by superposition of laser beams
Fundamentals
The Tilted Bose Hubbard Model (BHM)

\[ \mathcal{H} = -t \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) + \frac{U}{2} \sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j \]

The Tilted Bose Hubbard Model (BHM)

\[ \mathcal{H} = -t \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) + \frac{U}{2} \sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j - E \sum_j \mathbf{e} \cdot \mathbf{r}_j \hat{b}_j^\dagger \hat{b}_j \]

The Tilted Bose-Hubbard Model (BHM)

The Tilted Bose Hubbard Model (BHM)

Mott insulator

- Minimize $\mathcal{H}$ in limit $U \gg t$
- Average site occupation $n_0$

The Tilted Bose Hubbard Model (BHM)

Mott insulator

- Tunneling generates new state
- Energy difference $U - E$

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Define dipole operator

\[ \hat{d}_j^\dagger = \frac{\hat{b}_j \hat{b}_{j+1}^\dagger}{\sqrt{n_0(n_0 + 1)}} \]

with the conditions

\[ \hat{d}_j^\dagger \hat{d}_j \leq 1 \]

\[ \hat{d}_j^\dagger \hat{d}_j \hat{d}_{j+1} \hat{d}_{j+1}^\dagger = 0 \]

The Tilted Bose Hubbard Model (BHM)

\[ \mathcal{O}(t^2) \]

Dipole of length 2

The Tilted Bose Hubbard Model (BHM)

Rewrite Hamiltonian

\[ \mathcal{H} = -t \sum_{\langle i,j \rangle} (\hat{b}_{i}^\dagger \hat{b}_{j} + \hat{b}_{j}^\dagger \hat{b}_{i}) + \frac{U}{2} \sum_{j} \hat{b}_{j}^\dagger \hat{b}_{j}^\dagger \hat{b}_{j} \hat{b}_{j} \]

\[ - E \sum_{j} \mathbf{e} \cdot \mathbf{r}_{j} \hat{b}_{j}^\dagger \hat{b}_{j} \]

with dipole operators

\[ \mathcal{H}_{d} = -t \sqrt{n_{0}(n_{0} + 1)} \sum_{j} (\hat{d}_{j} + \hat{d}_{j}^\dagger) \]

\[ + (U - E) \sum_{j} \hat{d}_{j}^\dagger \hat{d}_{j} \]
The Tilted Bose Hubbard Model (BHM)

Mott Insulator (MI)  Broken symmetry phase (BSP)

Tilt increase

The Tilted Bose Hubbard Model (BHM)

Eigenenergies of $\mathcal{H}_d$ depend on $n_0$ and $\lambda = \frac{U-E}{t}$

$N = 8$, $n_0 = 1$

The Tilted Bose Hubbard Model (BHM)

Motivation

Fundamentals

Tilted BHM

Ising model

mapping

Experimental realization

Setup

Results

Conclusions

Scaling $\Delta E \propto N^{-z} = N^{-1}$ at $\lambda_c$

Ising model mapping

No tunneling event spin ↑
Tunneling event spin ↓

\[
S_j^z = \frac{1}{2} - \hat{d}_j^\dagger \hat{d}_j
\]
\[
S_j^x = \frac{1}{2} \left( \hat{d}_j^\dagger + \hat{d}_j \right)
\]
\[
S_j^y = \frac{i}{2} \left( \hat{d}_j^\dagger - \hat{d}_j \right)
\]

Ising model mapping

Mott Insulator (MI)  Broken symmetry phase (BSP)

Tilt increase

Ising model mapping

Constrains

- $\hat{d}_j^\dagger \hat{d}_j \leq 1$
  Fulfilled by definition

- $\hat{d}_j^\dagger \hat{d}_j \hat{d}_{j+1}^\dagger \hat{d}_{j+1} = 0$
  Add term to $\mathcal{H}_d$

$$J \hat{d}_{j+1}^\dagger \hat{d}_{j+1} \hat{d}_j^\dagger \hat{d}_j = J \left( S_{z}^{j+1} - 1/2 \right) \left( S_{z}^{j} - 1/2 \right)$$
Ising model mapping

1D Ising chain with longitudinal and transverse field equivalent to 2D Ising model

\[ \mathcal{H}_S = J \sum_j (S^j_z S^{j+1}_z - h_x S^j_x - h_z S^j_z) \]

No analytical solution

Ising model mapping
Phase transition

\[ h_z = 1 - \Delta/J, \quad h_x = 2\sqrt{2} t/J \]

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Experimental realization
Experimental realization

Setup

Experimental realization

Setup

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Experimental realization
Setup

Spin chain

Atom position in tilted lattice

Single site readout (odd/even)

a $\Delta < 0$: paramagnet

b $\Delta \approx 0$

c $\Delta > 0$: antiferromagnet

d Spin mapping

Experimental realization

Results

Experimental realization

Results

\[ S_z^j = \frac{1}{2} p_{\text{odd}}^j \]

Experimental realization

Results

Experimental realization

Results

Conclusions
Conclusions

Summary

- MI phase
- Tilted BHM
- Set of resonant states in 1D
- MI ground state — BSP
- Mapping to Ising model
- Experimental results

Conclusions
Conclusions and outlook

- Experimental results in agreement with theory
- Optical lattices as a promising quantum simulator

- Higher dimensions
- Different lattice geometries

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