Tilted Bose-Hubbard Model

Phase transition in 1D

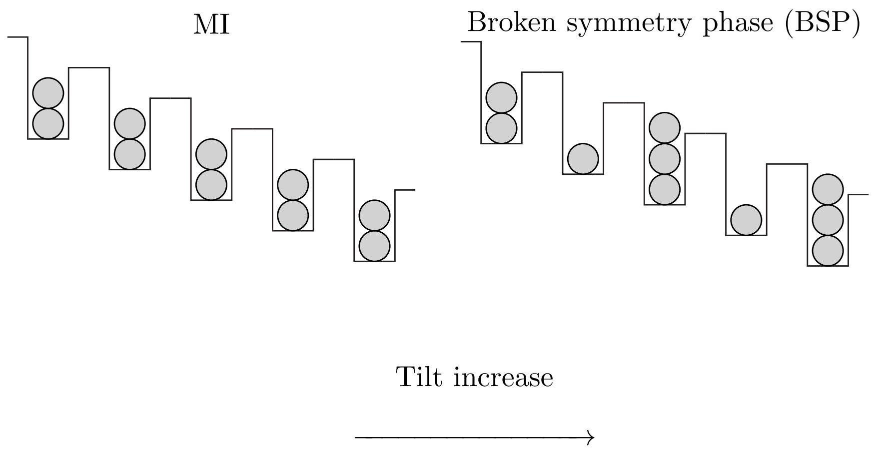
1 Fundamentals

- The Hamiltonian for the tilted Bose-Hubbard model (tilted BHM) reads

$$\mathcal{H} = \underbrace{-t\sum_{\langle i,j\rangle}(\hat{b}_i^\dagger\hat{b}_j + \hat{b}_j^\dagger\hat{b}_i)}_{\text{Tunneling}} + \underbrace{\frac{U}{2}\sum_j \hat{b}_j^\dagger\hat{b}_j^\dagger\hat{b}_j\hat{b}_j}_{\text{On-site interaction}} \underbrace{-E\sum_j \mathbf{e}\cdot\mathbf{r}_j\hat{b}_j^\dagger\hat{b}_j}_{\text{Tilt}}\,. \tag{1}$$

$\langle i,j\rangle$: Sum over nearest neighbours, $\hat{b}_i^\dagger$ ($\hat{b}_i$): Bosonic creation (annihilation) operator for the site i, t: Hopping term, U: On-site interaction, $\mathbf{r}_j$: Site position, $\mathbf{e}$: Vector along electric field

- For $U, E \gg t$, the system is in the Mott insulator (MI) phase
- The tilt can enhance tunneling



- A tunnel process results in a dipole, for which an operator

$$\hat{d}_j^\dagger = \frac{\hat{b}_j\hat{b}_{j+1}^\dagger}{\sqrt{n_0(n_0+1)}} \tag{2}$$

n_0: Mean site occupation of the MI

with the conditions

$$\hat{d}_j^\dagger\hat{d}_j \leq 1 \quad \text{and} \quad \hat{d}_j^\dagger\hat{d}_j\hat{d}_{j+1}^\dagger\hat{d}_{j+1} = 0 \tag{3}$$

can be defined.

- $\mathcal{H}$ can described with these operators as an effective Hamiltonian

$$\mathcal{H}_d = -t\sqrt{n_0(n_0+1)}\sum_j(\hat{d}_j + \hat{d}_j^\dagger) + (U-E)\sum_j \hat{d}_j^\dagger\hat{d}_j, \tag{4}$$

which only includes the dipole states and the ground state.

- The model can then be mapped to an Ising chain in transverse and longitudinal fields with the spin operators

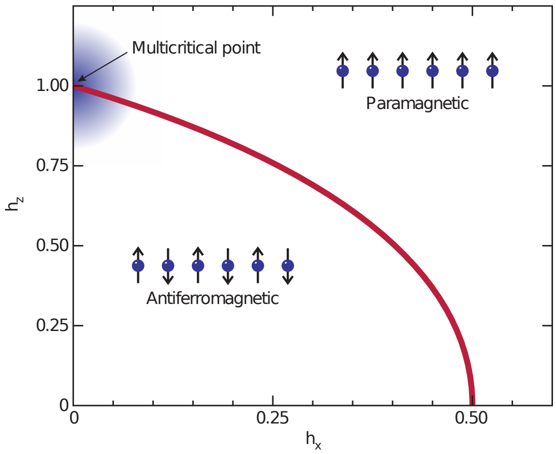
$$S_z^j = \frac{1}{2} - \hat{d}_j^\dagger \hat{d}_j, \quad S_x^j = \frac{1}{2}\left(\hat{d}_j^\dagger + \hat{d}_j\right), \quad S_y^j = \frac{i}{2}\left(\hat{d}_j^\dagger - \hat{d}_j\right) \tag{5}$$

- A new Hamiltonian of the form

$$\mathcal{H} = J \sum_j (S_z^j S_z^{j+1} - h_x S_x^j - h_z S_z^j) \tag{6}$$

  arises.

- The phase diagram for this Ising model is known:



2 Experimental realization

- Microscope with single-site readout
- $h_z \propto E$ ramped across transition
- Measurement of $p_{\text{odd}} \propto S_z$ shows phase transition and reversibility