Superfluidity and Condensation

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Hauptseminar Physics of cold Gases

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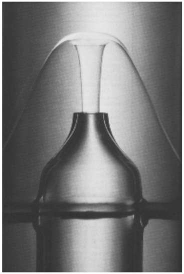
The discovery of superfluidity

- *Early 1930's*: Peculiar things happen in ^{4}He below the λ-temperature $T_\lambda = 2.17\,\mathrm{K}$
- *1938*: Kapitza, Allen & Misener measure resistance to the flow of liquid helium $\Rightarrow$ Superfluidity below T_λ
- *1938*: Fritz London - Superfluidity is related to Bose-Einstein condensation

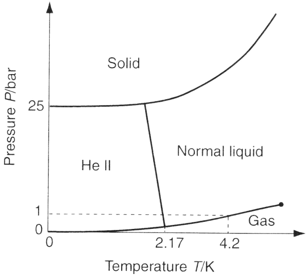


Fountain effect. [1]

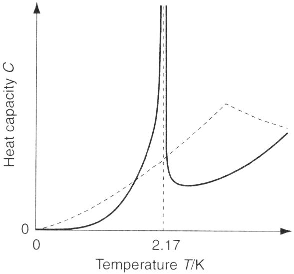
- *until 1941*: Laszlo Tisza and Lev Landau - two fluid model
- *1972*: Superfluidity in ^{3}He $\Rightarrow$ also in fermionic systems

[1] Nature 141, 243-244 (1938)

Superfluidity in ^{4}He



Phase diagram of ^{4}He. [2]



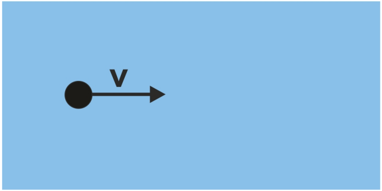
Heat capacity of ^{4}He. [2]

[2] from: Tony Guénault, Basic Superfluids

Outline

The Landau criterion

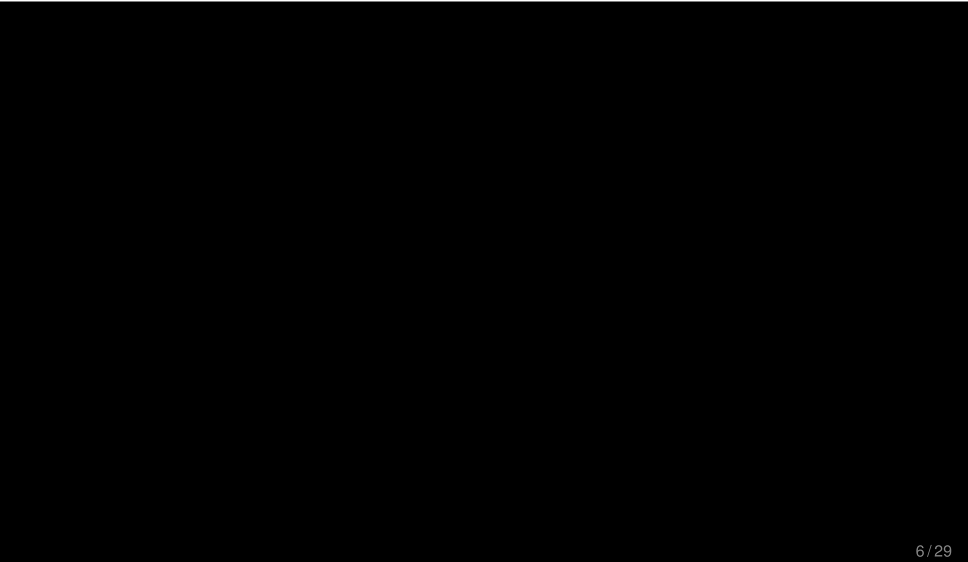
Where does the friction come from?



- Dissipation arises from elementary excitations
- At what velocity is it possible to create excitations?

The Landau criterion

What is the requirement to create an excitation?



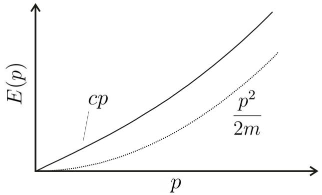
The Landau criterion

Landau critical velocity: $v_c = \min_{\mathbf{p}} \frac{E(\mathbf{p})}{p}$

- Ideal Bose gas: $v_c = 0\,\mathrm{ms}^{-1}$
  $\hookrightarrow$ no superfluidity!

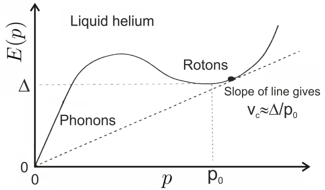
- Weakly interacting Bose gas:

$$E(\mathbf{p}) = \sqrt{\left(\frac{\mathbf{p}^2}{2m}\right)^2 + \frac{gn}{m}\mathbf{p}^2} \;\Rightarrow\; v_c \approx c$$



Ideal and weakly interacting Bose gas.

- Liquid helium: $v_c \approx 60\,\mathrm{ms}^{-1}$
  Dispersion relation from neutron scattering experiments

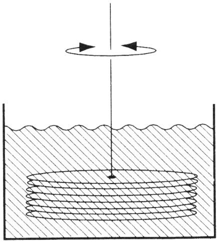


Liquid ^{4}He. [3]

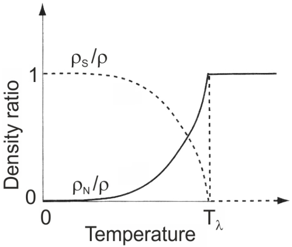
[3] from: Tony Guénault, Basic Superfluids (altered)

Two fluid model

Torsional oscillator: oscillation frequency $\omega = \sqrt{\frac{K}{I}}$



Andronikashvili's experiment. [4]



Experimental Results. [4]

$\Rightarrow$ Two fluid modell:

- Mass density: $\rho = nm = \rho_N + \rho_S$
- Viscosity: $\eta_N > 0$, $\eta_S = 0$

[4] from: Tony Guénault, Basic Superfluids

Two fluid model

Component densities

Can we give an explanation for the behavior of the component densities?

- Uniform fluid at finite, small temperature in a capillary
- Just thermal excitations
- Assume: gas of noninteracting excitations (quasiparticles) in thermal equilibrium
- Mass flow associated with quasiparticles is not superfluid

Two fluid model

Component densities

At equilibrium: mean velocity $\mathbf{v}_{\mathrm{N}}$ of excitations equals velocity of capillary

Relative velocity of superfluid and capillary:

$$\mathbf{v}_{\mathrm{S}} - \mathbf{v}_{\mathrm{N}}$$

Excitation energy in capillary frame:

$$E(\mathbf{p}) + \mathbf{p}(\mathbf{v}_{\mathrm{S}} - \mathbf{v}_{\mathrm{N}})$$

Equilibrium distribution of excitations:

$$f_{\mathbf{p}} = \frac{1}{\exp\left(\frac{E(\mathbf{p})+\mathbf{p}\cdot(\mathbf{v}_{\mathrm{S}}-\mathbf{v}_{\mathrm{N}})}{kT}\right) - 1}$$

Two fluid model

Component densities

Mass density: $\rho = nm = \rho_S + \rho_N$

Mass current: $m\mathbf{j} = \rho_S \mathbf{v}_S + \rho_N \mathbf{v}_N$

Total momentum carried by the fluid:

$$\mathbf{P} = M\mathbf{v}_S + \sum_i \mathbf{p}_i \qquad \Rightarrow \qquad m\mathbf{j} = \rho \mathbf{v}_S + \int \frac{d^3p}{(2\pi\hbar)^3} \mathbf{p} f_{\mathbf{p}}$$

$$\rho_N(\mathbf{v}_N - \mathbf{v}_S) = \int \frac{d^3p}{(2\pi\hbar)^3} \mathbf{p} f_{\mathbf{p}}$$

For small relative velocities $\mathbf{v}_S - \mathbf{v}_N$:

$$\rho_N = -\frac{1}{3} \int \frac{d^3p}{(2\pi\hbar)^3}\, p^2\, \frac{df_{\mathbf{p}}(E, \mathbf{v}_S - \mathbf{v}_N = 0)}{dE}$$

Two fluid model

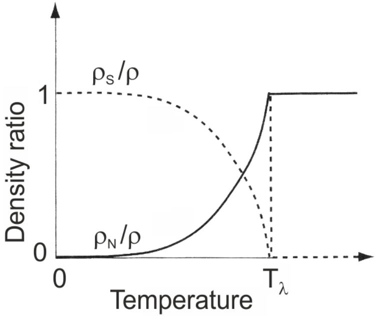
Component densities

Low temperatures:
just phonon part contributes

$$E(p) = cp$$

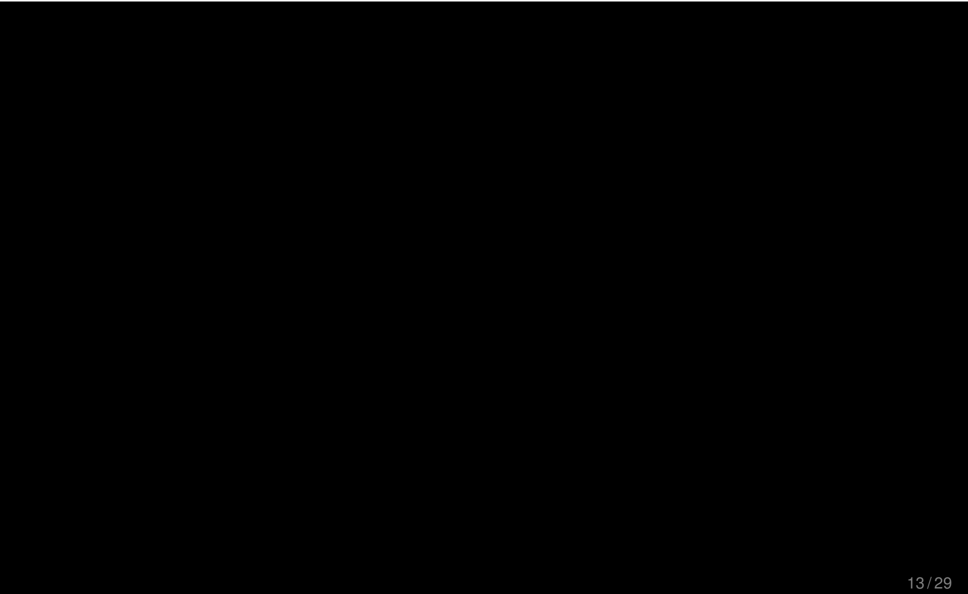
$$\Downarrow$$

$$\rho_{\mathrm{N}} = \frac{2\pi^2 (kT)^4}{45\hbar^3 c^5}$$



from: Tony Guénault, Basic Superfluids

Off-diagonal long-range order



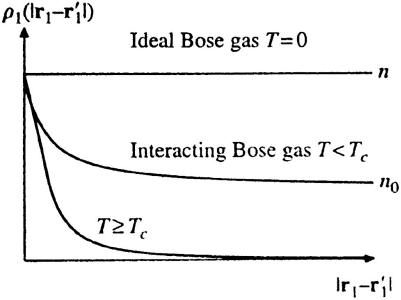
Off-diagonal long-range order

One particle density matrix describes correlation between the particles:

$$\rho_1(\mathbf{r}-\mathbf{r}') = \langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}')\rangle$$

- $|\mathbf{r}-\mathbf{r}'| \longrightarrow 0: \quad \rho_1(\mathbf{r}-\mathbf{r}') \longrightarrow n$
  $|\mathbf{r}-\mathbf{r}'| \longrightarrow \infty: \quad \rho_1(\mathbf{r}-\mathbf{r}') \longrightarrow n_0$

  $\hookrightarrow$ Off-diagonal long-range order
- Calculation via quantum Monte Carlo method
- Liquid helium:
  $n_0(T=0) \approx 0.1n$
  $n_0(T > T_\lambda) \approx 0$

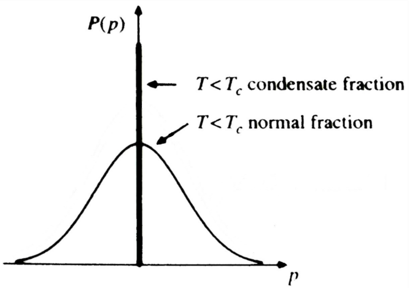


from: J. F. Annett, Superconductivity, Superfluids, and Condensates

Off-diagonal long-range order

Momentum distribution

$$N_{\mathbf{k}} = \langle a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} \rangle = N_0 \delta_{\mathbf{k},\mathbf{0}} + \widetilde{N}(\mathbf{k})$$



from: J. F. Annett, Superconductivity, Superfluids, and Condensates (altered)

$\Rightarrow$ At T=0 the superfluid density approaches ρ but just 10% of the helium is condensed into the groundstate!

Off-diagonal long-range order

Order parameter

Points are statistically independent for $|\mathbf{r}-\mathbf{r}'| \to \infty$:

$$\langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}')\rangle \;\to\; \langle\hat{\psi}^\dagger(\mathbf{r})\rangle\langle\hat{\psi}(\mathbf{r}')\rangle$$

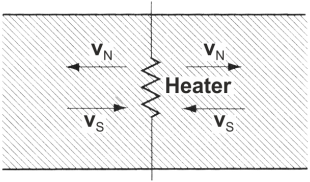
Order parameter of the system:

$$\psi_0(\mathbf{r}) = \langle\hat{\psi}(\mathbf{r})\rangle = \sqrt{n_0}e^{i\theta}$$

Thermodynamic effects

Consequences of the two fluid model

- Entropy is carried by normal component: $S_S = 0$
  $\hookrightarrow$ heat transport exclusively due to ρ_N
- very efficient heat transport (frictionless counterflow)



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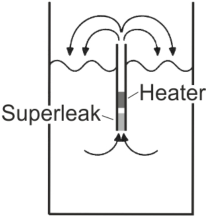
Fountain effect:



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Quantization of flow

Order parameter:

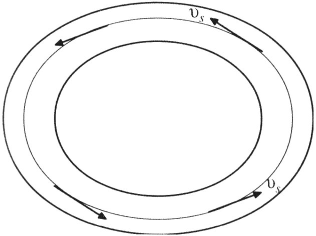
$$\psi_0 = \sqrt{n_0} e^{i\theta}$$

Superfluid velocity:

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \theta$$

Irrotational flow:

$$\nabla \times \mathbf{v}_s = 0$$



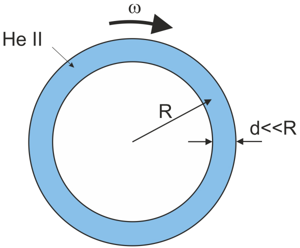
from: J. F. Annett, Superconductivity, Superfluids and Condensates

Circulation:

$$\kappa = \oint d\mathbf{r}\, \mathbf{v}_s = \frac{\hbar}{m} \oint d\mathbf{r}\, \nabla \theta = \frac{\hbar}{m} \Delta \theta$$

Quantisation: $\Delta \theta = 2\pi n \quad \Rightarrow \quad \kappa = \frac{h}{m} n$

Quantization of flow



$$\oint d\mathbf{r}\, \mathbf{v}_s = \frac{h}{m} n$$

for n=1: $2\pi R \cdot \omega_c R = \frac{h}{m}$

$$\hookrightarrow \omega_c = \frac{\hbar}{mR^2}$$

- Rotate slowly with ω at $T > T_\lambda$
  Fluid velocity: $v = \omega R$
  Moment of inertia: $I_{cl} = NmR^2$
- Cool through T_λ
  $\omega_N = \omega$
  $\omega_S = \frac{\hbar}{mR^2} n$
  n takes value closest to ω / ω_c
  $\hookrightarrow \omega << \omega_c : I(T) < I_{cl}$

Vortices

- In cylindrical coordinates:

  $\nabla \times \mathbf{v}_s = 0 \;\; \forall\, r \neq 0$

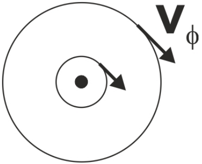
  if $\frac{1}{r}\frac{\partial}{\partial r}(r v_\Phi) = 0$

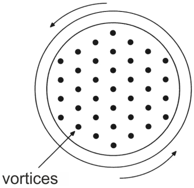
  $\hookrightarrow \;\; \mathbf{v}_s = \frac{\kappa}{2\pi r}\mathbf{e}_\Phi$

- No singularities in $\mathbf{v}_s$
  $\Rightarrow \;\; \psi_0$ vanishes at $r = 0$

- Vortex cores in helium: ≈ 1Å

  Visualization: tracer particles





from: J. F. Annett, Superconductivity, Superfluids and Condensates

First & second sound

Superfluid: Two degrees of freedom associated with normal and superfluid component $\Rightarrow$ two sound-like modes

Coupled wave equations:

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p$$

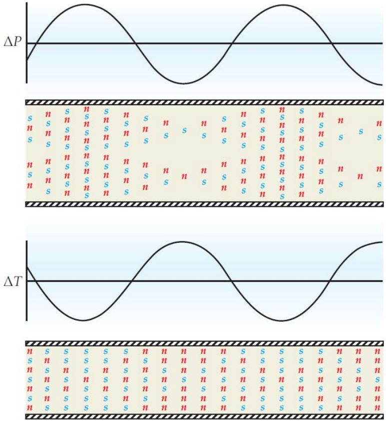
$$\frac{\partial^2 \widetilde{s}}{\partial t^2} = \frac{\rho_s \widetilde{s}^2}{\rho_n} \nabla^2 T$$

Sound velocities:

$$c_1^2 = \frac{\partial p}{\partial \rho}, \quad c_2^2 = \frac{\rho_s T \tilde{s}^2}{\rho_n \tilde{c}}$$

- First sound: pressure/density wave
- Second sound: temperature/entropy wave

First & second sound



from: Phys. Today 62, 10, 34 (2009)

First sound:

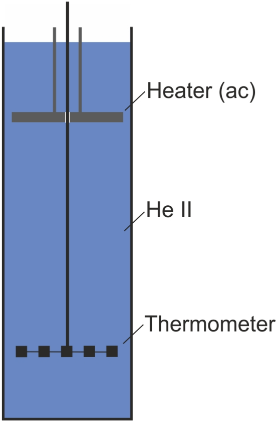
- Density variations driven by pressure variations
- Two components in phase

Second sound:

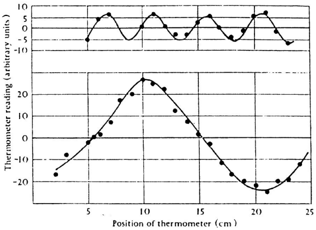
- Density constant
- Composition of density varies
- Entropy variations driven by temperature variations
- Two components counter-oscillate

Second sound

Experiment by Peshkov (1944)



- Ac current creates temperature variations
- Glass tube acts as resonator
  $\hookrightarrow$ standing wave
- Standing wave patterns are traced with a movable thermometer



from: R. Srinivasan: "Second Sound: Waves of Entropy and Temperature". Resonance 3: 16–24

Supersolids

Observation of Superflow in Solid Helium

E. Kim and M. H. W. Chan*

We report on the observation of nonclassical rotational inertia in solid helium-4 confined to an annular channel in a sample cell under torsional motion, demonstrating superfluid behavior. The effect shows up as a drop in the resonant oscillation period as the sample cell is cooled below 230 millikelvin. [...]

This experiment indicates that superfluid behavior is found in all three phases of matter.

Science 24 September 2004: 305 (5692), 1941-1944

- Superfluid behavior in crystal
- Off-diagonal + diagonal long-range order
- Non-classical rotational inertia

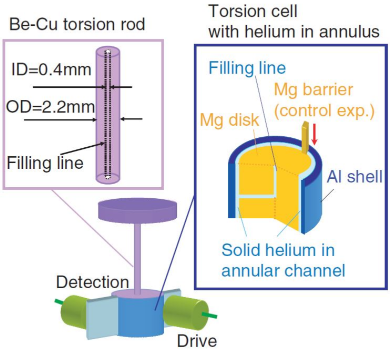
Supersolids

Torsional oscillator experiment

- Superfluid fraction decouples from oscillation
- Reduced moment of inertia:

$$I(T) = I_{\text{classical}} \cdot \frac{\rho_n(T)}{\rho}$$

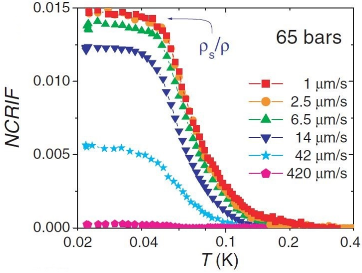
- Resonance frequency: $\omega_r = \sqrt{\frac{K}{I}}$



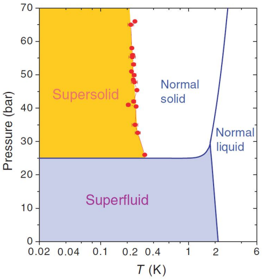
from: Science 24 September 2004: 305 (5692), 1941-1944

Supersolids

Experimental results



Non-Classical Rotational Inertia Fraction. [5]



Phase diagram. [5]

[5] from: Science 24 September 2004: 305 (5692), 1941-1944

Supersolids

Models and problems

Models:

- Zero motion creates a gas of zero-point vacancies undergoing BEC
  $\hookrightarrow$ superfluidity of vacancies = superflow of particles
- Exchange processes between neighboring atoms

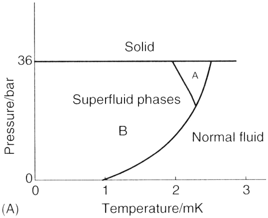
But: Not clear if interpretation of experiments is correct!

$\hookrightarrow$ existence of supersolids in nature not verified

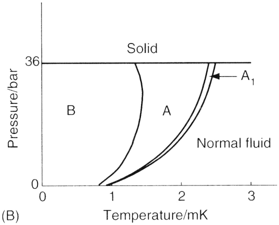
Superfluidity in ^{3}He

- ^{3}He: fermion
- $T_c \approx 2.5\,\mathrm{mK}$
- Phase diagram:

without B-field: [6]



with B-field: [6]



Superfluid state linked with condensation ⇒ pairing mechanism

[6] from: Tony Guénault, Basic Superfluids

Summary

- Superfluidity below Landau critical velocity v_c
- Two fluid model: $\rho = \rho_N + \rho_S$
- Off-diagonal long-range order
- Irrotational flow of superfluid $\Rightarrow$ vortices, NCRI
- First & second sound: counter-oscillating ρ_N and ρ_S
- Supersolids: superfluid behavior in solids, NCRI
- Superfluidity in ^{3}He $\Rightarrow$ pairing mechanism