Superfluidity and Condensation

Hauptseminar: Physics of cold Gases
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1 The Landau criterion

- Friction: loss of kinetic energy due to dissipation
- Dissipation from elementary excitations with dispersion $E(p)$
- If a fluid moves below the Landau critical velocity

$$v_{\mathrm{c}} = \min_{\mathbf{p}} \frac{E(\mathbf{p})}{p}$$

  relative to an obstacle, no excitations can be created → superfluidity.
- Elementary excitations in liquid ^{4}He: phonons + rotons (fig. 2.1)

2 Two fluid model

- Superfluid consists of two interpenetrating fluids (normal and superfluid component)
- Mass density: $\rho = \rho_{\mathrm{n}} + \rho_{\mathrm{s}}$ Viscosity: $\eta_{\mathrm{n}} > 0,\ \eta_{\mathrm{s}} = 0$ Entropy: $S_{\mathrm{s}} = 0$
- Linear dispersion $E(p) = c \cdot p \rightarrow \rho_{\mathrm{N}} = \frac{2\pi^2 (kT)^4}{45\hbar^3 c^5}$ (fig. 2.2)

3 Off-diagonal long-range order

- One particle density matrix: $\rho_1(\mathbf{r} - \mathbf{r}') = \langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}') \rangle$
- $|\mathbf{r} - \mathbf{r}'| \longrightarrow 0:\ \rho_1(\mathbf{r} - \mathbf{r}') \longrightarrow n$ (density)
- $|\mathbf{r} - \mathbf{r}'| \longrightarrow \infty:\ \rho_1(\mathbf{r} - \mathbf{r}') \longrightarrow n_0$ (groundstate density)
- Liquid helium: Monte Carlo Methods → $n_0 \approx 0.1 \cdot n$
  $\hookrightarrow$ ρ_{s} not equivalent to condensate fraction!

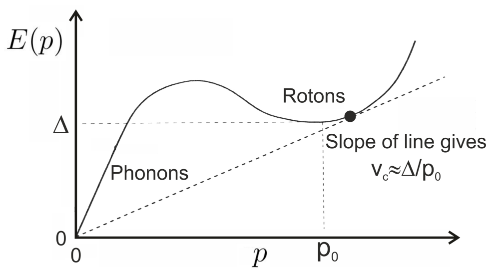

Figure 2.1: Dispersion relation for excitations in liquid ^{4}He. From: Tony Guénault, Basic Superfluids (altered).

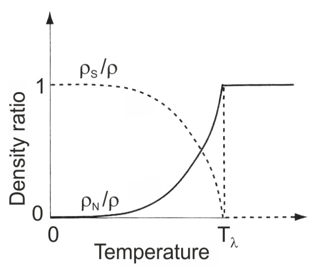

Figure 2.2: Temperature dependence of the normal and superfluid component. From: Tony Guénault, Basic Superfluids.

4 Quantization of flow

- Order parameter: $\psi_0(\mathbf{r}) = \langle \hat{\psi}(\mathbf{r}) \rangle = \sqrt{n_0} e^{i\theta}$
- Superfluid velocity: $\mathbf{v}_\mathrm{s} = \frac{\hbar}{m} \nabla \theta$
- Irrotational flow: $\nabla \times \mathbf{v}_\mathrm{s} = 0$
- Circulation: $\kappa = \oint d\mathbf{r}\, \mathbf{v}_\mathrm{s} = \frac{\hbar}{m} \oint d\mathbf{r}\, \nabla \theta = \frac{\hbar}{m} \Delta \theta$
- Quantization: $\Delta \theta = 2\pi n \qquad \Rightarrow \qquad \kappa = \frac{\hbar}{m} n$
- Angular velocity of superfluid component below T_λ: $\omega_\mathrm{S} = \frac{\hbar}{mR^2} n$
  $\hookrightarrow$ rotate slowly with ω: $n = 0 \rightarrow$ superfluid component decouples from motion
  $\hookrightarrow$ reduced moment of inertia: $I(T) = I_\mathrm{classical} \cdot \frac{\rho_\mathrm{n}(T)}{\rho}$

5 First & second sound

First sound:

- Density variations driven by pressure variations
- Two components in phase

Second sound:

- Density constant
- Composition of density varies $\rightarrow$ two components counter-oscillate
- Entropy variations driven by temperature variations

6 Supersolids

- Superfluid behavior in the solid phase
- Off-diagonal + diagonal long-range order
- Observation of NCRI in solid ^{4}He: $I(T) = I_\mathrm{classical} \cdot \frac{\rho_\mathrm{n}(T)}{\rho}$
- Existence in nature not verified yet

7 Superfluidity in ^{3}He

- ^{3}He is a fermion $\rightarrow$ pairing mechanism analog to Cooper pairs in BCS-Theorie
- Phase diagram:

(a) without B-field

(b) with B-field

Figure 7.1: Phase diagram of ^{3}He (a) without and (b) with external B-field. From: Tony Guénault, Basic Superfluids.