Superfluidity in 2D: Kosterlitz Thouless Phase Transition

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In 1966 Mermin and Wagner proved the absence of long-range order in two-dimensional systems with a continuous symmetry (e.g. XY-model) for finite temperature T [1]. Though, Kosterlitz and Thouless showed the existence of a phase transition in 1972. A phase transition is produced by unbinding vortex-antivortex pairs at a critical temperature T_{KT} [2].

1 XY-Model

Two dimensional square lattice with planar rotors of unit length.

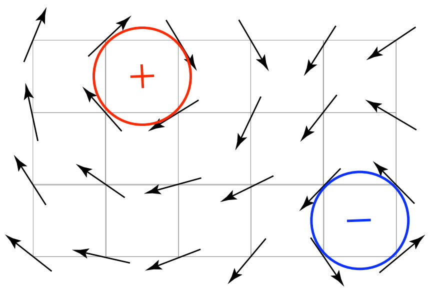

Figure 1: Schematic XY model

Find Hamiltonian H and interaction term J:

$$H = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) \tag{1}$$

For low temperature with ground state energy $E_0 = 2JN$:

$$H = E_0 + \frac{J}{2} \int d^2r (\nabla\theta)^2 \tag{2}$$

Free energy F of a single vortex with system size L and lattice spacing a:

$$F = E_0 + (\pi J - 2k_B T) \ln(L/a) \tag{3}$$

Indentify critical temperatur $T_{KT} = \frac{\pi J}{2k_B}$

2 Phase Transition

2.1 Mermin Wagner Theorem

Looking at the mean magnetisation $\langle \mathbf{S} \rangle$ Mermin and Wagner showed that there is no long-range order to be found in 2D systems (e.g XY-model) for finite temperature. $\langle \mathbf{S} \rangle$ goes to zero for finite temperature. Fluctuations at even low temperature will destroy the long-range order.

2.2 Correlation Function

Looking at the correlation function $\langle \mathbf{S}(\mathbf{r})\mathbf{S}(\mathbf{0}) \rangle$, you will find quasi-long-range order:

- Low temperature:

$$\langle \mathbf{S}(\mathbf{r})\mathbf{S}(0) \rangle \simeq \left(\frac{r}{L}\right)^{-T/2\pi J} \tag{4}$$

$\rightarrow$ quasi-long-range order.

- High temperature

$$\langle \mathbf{S}(\mathbf{r})\mathbf{S}(0) \rangle = e^{-\frac{r}{\xi}} \tag{5}$$

$\rightarrow$ exponential decay.

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2.3 Superfluid Density

Superfluid density ρ_s measures the energy change caused by a twist. Therefore it describes the effect of thermally activated vortex pairs.

$$\rho_s^R = \begin{cases} \rho_s^R(T_{KT}^-)[1 + const.(T_{KT} - T)^{1/2}] \\ 0 \end{cases} \quad (6)$$

with the first case showing $T < T_{KT}$ and second case $T > T_{KT}$.

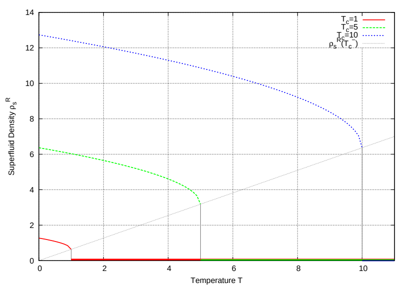

Figure 2: Superfluid density over T.

Universal for all systems you get $\rho_s^R(T_{TK}^-)/T_{KT} = 2/\pi$.

3 Kosterlitz-Thouless Phase Transition

Increasing the temperature activates vortices. For $T < T_{KT}$ activated vortices bind to vortex-antivortex pairs. At T_{KT} all pairs unbind collective.

3.1 Experiment

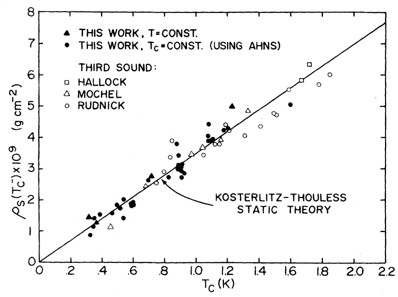

Figure 3: Experiment with superfluid helium films.

References

[1] N.D. Mermin and H. Wagner, Phys. Rev. Lett. 22 1133 (1966).

[2] J.M. Kosterlitz and D.J. Thouless, J. Phys. C 6, 1181 (1973).

[3] P.M. Chaikin and T.C. Lubensky, *Principles of condensed matter physics*, Cambridge University Press (1995).

[4] H. J. Jensen, *The Kosterlitz-Thouless Transition*, Department of Mathematics, Imperial College (2003).