BEC in one dimension

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1 One-dimensional BEC - important parameters

The atomic motion is cooled down below the transverse vibrational energy $\hbar\omega_\perp$, but the motion in z-direction is free.
$\Rightarrow$ condition for 1D: $k_B T \ll \hbar\omega_\perp$.
Reduced dimensionality leads to an absence of long-range order.
$\Rightarrow$ quasi-condensate

1.1 Description as Luttinger Liquid

Describe the low energy 1D fluid as a Luttinger Liquid with the local fluctuation field $\Pi(x)$ and the phase $\varphi(x)$ the Hamiltonian can be obtained as

$$H \approx \frac{\hbar^2}{2m} \int dx \left[v_J \left(\Delta\varphi(x)\right)^2 + v_N \left(\Delta\Pi(x)\right)^2 \right]$$

with the sound velocity of density fluctuations $v_s = \sqrt{v_N v_J} = \sqrt{\frac{\kappa}{m\rho_0}}$.
$\Rightarrow$ in 1D there are always fluctuations which destabilize the BEC. correlation function:

$$\left\langle \Psi^\dagger(x)\Psi(0) \right\rangle \propto x^{-\frac{1}{\eta}}$$

with the correlation exponent $\eta = 2\sqrt{\frac{v_J}{v_N}}$.
In 1D the correlation function decays algebraically in the size of the system.

1.2 Different regimes in a one-dimensional BEC

At low temperatures, there are different regimes described by the parameter γ:

$$\gamma = \frac{m g_{1D}}{\hbar^2 n_{1D}}$$

with the coupling strength g_{1D} and the 1D-density n_{1D}.
Here, low density corresponds to the high interacting regime, which is the reverse in 3D.

- BEC ($\gamma \ll 1$, weakly interacting regime)
- quasi-condensate
- Tonks-Girardeau gas ($\gamma \gg 1$, strongly interacting regime)

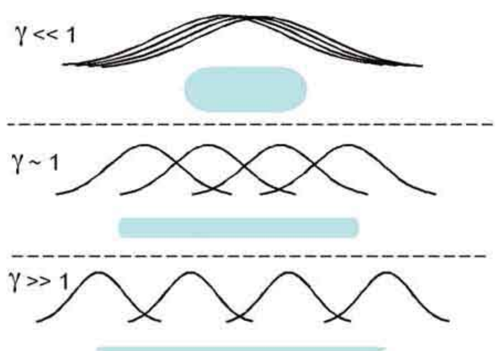

fig. cartoon of 1D atom distributions. In the strongly interacting regime (TG-regime), the single particle wave functions are spatially distinct.

2 Theoretical description

2.1 Tonks-Girardeau gas, $\gamma \to \infty$

Condition, that the interparticle interactions have an impenetrable core: $\Psi(x_1,...,x_n) = 0$, if $x_i = x_j$.

$$\Rightarrow \Psi_0^B = \left|\Psi_0^F\right|$$

2.2 How is the interaction?

Pseudo-potential

$$U_p(r) = g\delta(r)\left(\frac{\partial}{\partial r}r\right)$$

For low velocities the scattering amplitude can be approximated to $f(k_z) = -\frac{1}{1+ik_z a_{1D}}$ for the one-dimensional δ-potential

$$U_{1D}(z) = g_{1D}\delta(z) \qquad \text{with} \qquad g_{1D} = -\frac{\hbar^2}{m a_{1D}}$$

With this coupling strength g_{1D} the potential U_{1D} produces the same phase shift as the pseudo-potential U_P $\Rightarrow$ now it is possible to have a pure one-dimensional description.

2.3 Exact analysis of the interacting bose gas by Lieb and Liniger

The Schrödinger equation for N particles reads

$$\left(-\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g_{1D}\sum_{<i,j>}\delta(x_i - x_j)\right)\Psi = E\Psi$$

Using the Bethe ansatz

$$\Psi(x_1,\ldots,x_N) = \sum_P a(P)e^{i\sum_n k_{P(n)}x_n}$$

this problem can be solved exactly. Properties like the chemical potential μ or the potential and kinetic energy per particle v and t can be calculated (figure). For $\gamma \to \infty$ the properties show fermi-like values and the system is in the Tonks-Girardeau regime.

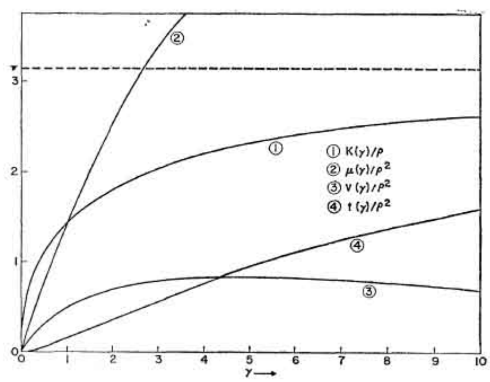

FIG. 1. Various numerically derived properties of the ground state plotted as functions of $\gamma = c/\rho$. K=cutoff momentum, μ=chemical potential, v=potential energy per particle, and t=kinetic energy per particle. As $\gamma \to \infty$: $K \to \pi\rho$; $\mu \to \pi^2\rho^2$, $v \to 0$, and $t \to \frac{1}{3}\pi^2\rho^2$.

3 Experimental relaization

Two independent light traps: blue-detuned crossed beam pairs confine nearly zero temperature ^{87}Rb atoms in an array of parallel tubes (figure). The red-detuned trap weakly confines them along the tubes. So it is possible to measure the BEC for different values of γ and the gas can be made either BEC-like or TG-like.

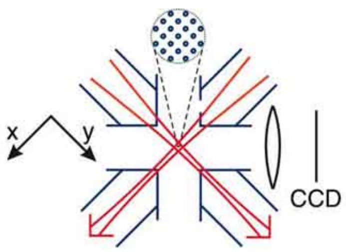

fig: scheme illustrating the experiment. the blue-detuned crossed beam pairs form the 2D lattice that strongly confines atoms in 1D tubes. The red-detuned beams trap the atoms axially