BCS-BEC Crossover

Hauptseminar: Physik der kalten Gase

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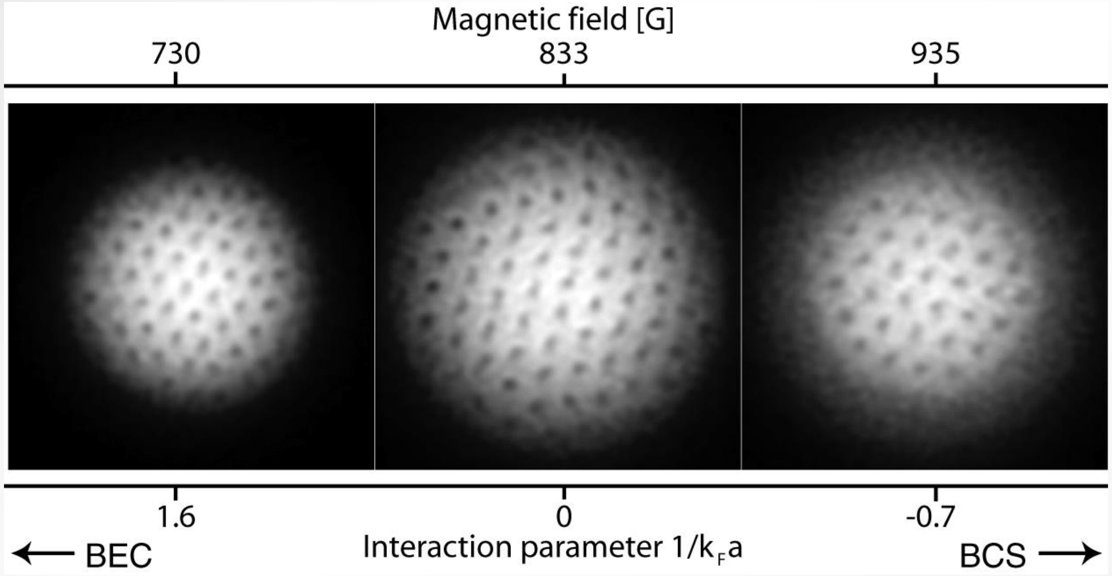
Outline

- Motivation
- Cold fermions
- BCS-Theory
  - Gap equation
- Feshbach resonance
- Pairing
- BEC of molecules
- BCS-BEC-crossover
- Conclusion

Motivation

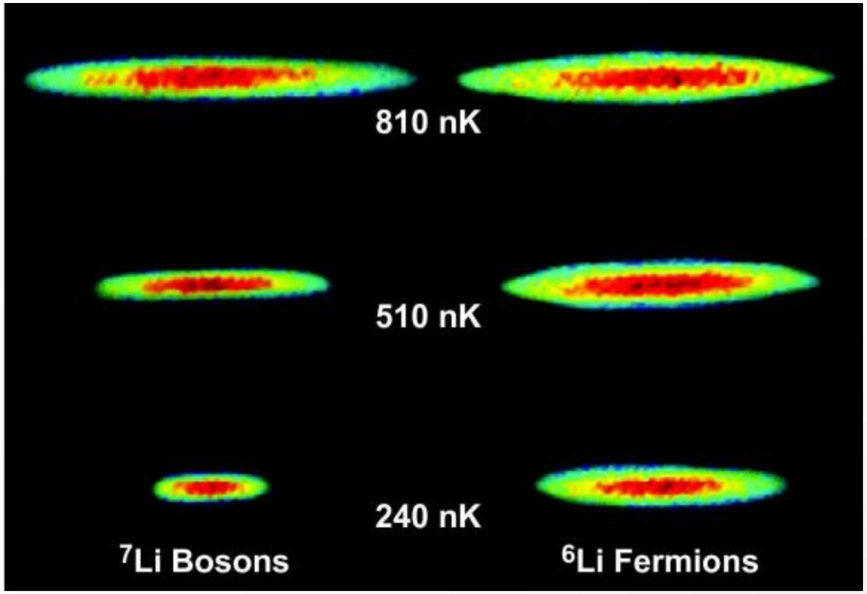


Source: Ketterle, Nature 435, 1047 (2005)

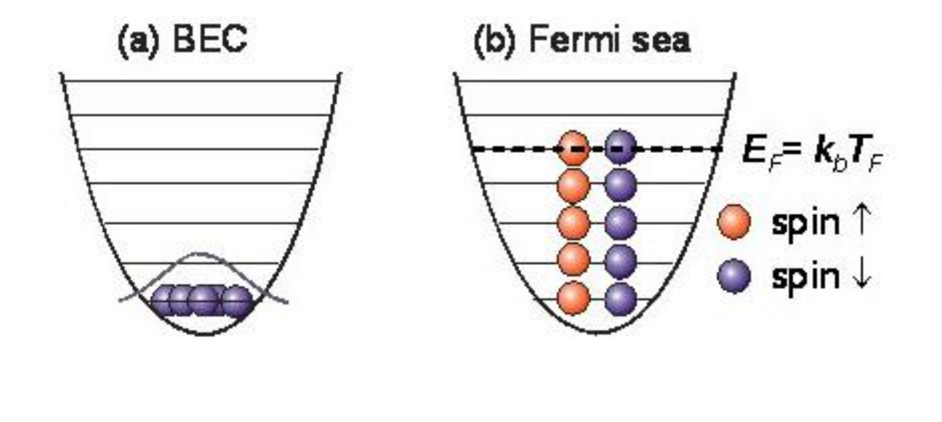
- Any connection between superfluid phases?
- Feshbach resonance is a very powerful tool for investigation

Cold fermions

- Fermi-Sea
  - For T=0 all states for $E \leq E_F$ are filled up
- Fermi-pressure prevents fermionic gas to shrink any further

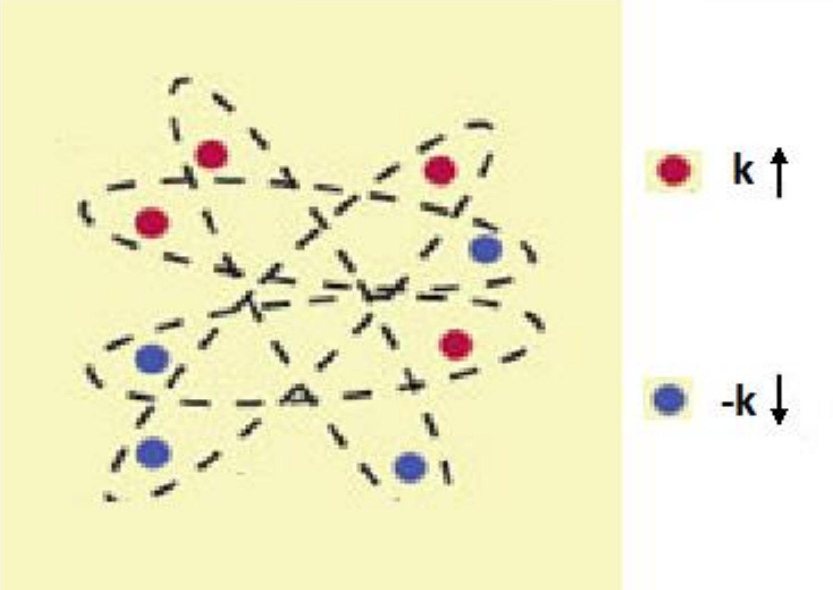


Source: http://arxiv.org/pdf/0704.3011



BCS-Theory

- Superconductivity was discovered in 1911
- Bardeen, Cooper, Schriefer developed a theory in 1957, that described conventional Superconductivity (SC)
- Electrons form Cooper pairs (Cp)



BCS-Theory

- Cp described in momentum space
- Wavefunction of Cp $\sim 1\mu m$ in real space
- Interaction via virtual phonons (lattice deformations)
- Cp are bosons → Condensation
- Fermi-Sea unstable against formation of pairs

BCS-Groundstate

- Single Cp Wavefunction:

$$|\psi_1\rangle = g(k) c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} |\phi_0\rangle$$

- Most general N-electron wavefunction

$$|\psi_N\rangle = \sum_{k_i \dots k_l} g(k_i, \dots, k_l) c^\dagger_{k_i\uparrow} c^\dagger_{-k_i\downarrow} \dots c^\dagger_{k_l\uparrow} c^\dagger_{-k_l\downarrow} |\phi_0\rangle$$

- $|\phi_0\rangle$: vacuum state
- g: weighting factor

BCS-Groundstate

- Very hard to solve
- Description of BCS-Groundstate differently
- Mean-field approach
- Occupancy of each state depends solely on average occupancy of other states

BCS-Groundstate

- Groundstate described by:

$$|\psi_G\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|\phi_0\rangle$$

- $|u_k|^2$: probability of no Cp being formed
- $|v_k|^2$: probability of Cp being formed
- Where $|u_k|^2 + |v_k|^2 = 1$

BCS-Theory

- Hamiltonian for interacting fermions:

$$H = \sum_{\sigma} \sum_{k} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_{kk'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

[kinetic energy] [interaction]

- $V_{kk'} > 0$
- Zero total momentum

BCS-Theory

- Particle number has to be regulated
- $H' = H - \mu N$
- $\epsilon_k \Rightarrow \xi_k = \epsilon_k - \mu$
- Two different ways of deriving the gap equation
- Variational method:

$$\delta \langle \psi_G | H' | \psi_G \rangle = 0$$

BCS-Theory

- Solution by canonical transformation
- Hamiltonian is not diagonalized
- Bogoliubov like transformation
- Quasiparticle operators:

$$c_{k\uparrow} = u_k^* a_{k0} + v_k a_{k1}^\dagger$$
$$c_{-k\downarrow} = -u_k^* a_{k0} + u_k a_{k1}^\dagger$$

BCS-Gap

$$H = \sum_{\sigma} \sum_{k} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_{kk'} V_{kk'} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

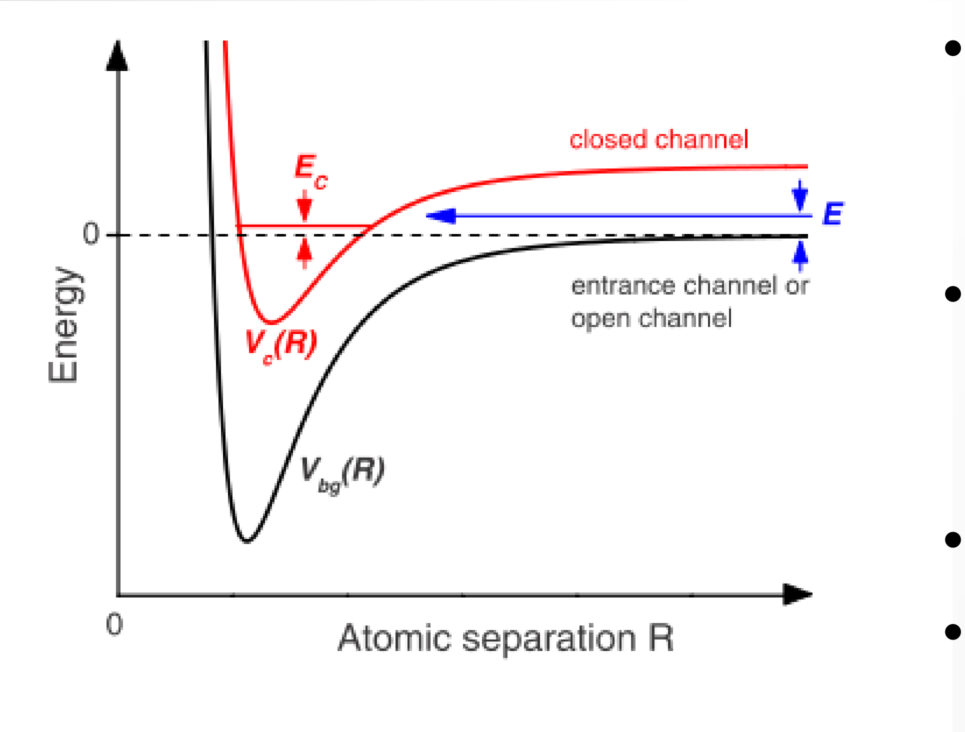
Summary Calculation

- Gap vanishes if normal state is reached
- Gap is order parameter of the system

$$\Delta = 2\hbar\omega_D e^{-\frac{1}{\lambda N(0)}}$$

- λ: interaction between phonon (crystal) and electrons
- ω_D: phonon energy scale
- N(0): free-electron energy scale

Feshbach resonance



Source: Rev. Mod. Phys. 82, 1225–1286 (2010)

c

- Two atoms in open channel state can couple to a closed channel state
- Bound state of molecule is close to the threshold of background potential
- Coupling via magnetic field
- Atoms in different hyperfine states

Feshbach resonance

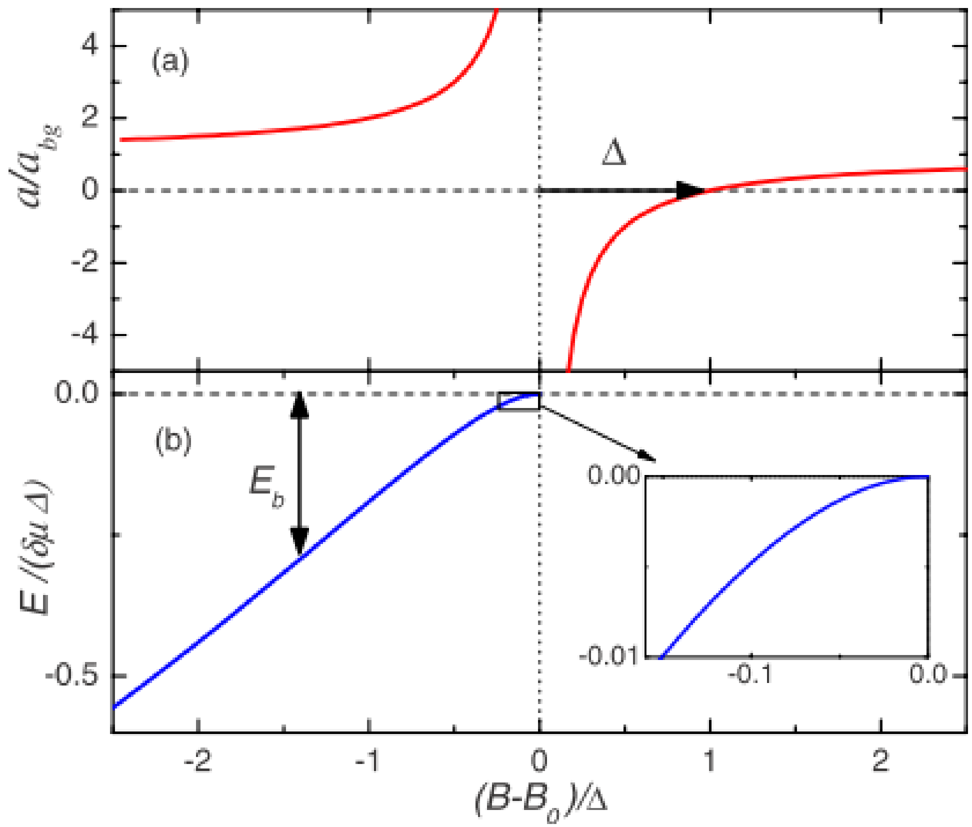
- Cold regime:
  - scattering length a
- Actually the phase shift φ after the scattering process is essential
- $\tan\varphi = -ka$
- For $a > 0$:

  $\varphi < 0$ repulsive regime
- For $a < 0$

  $\varphi > 0$ attractive regime

Forming molecules

BEC BCS



Source: Rev. Mod. Phys. 82, 1225–1286 (2010)

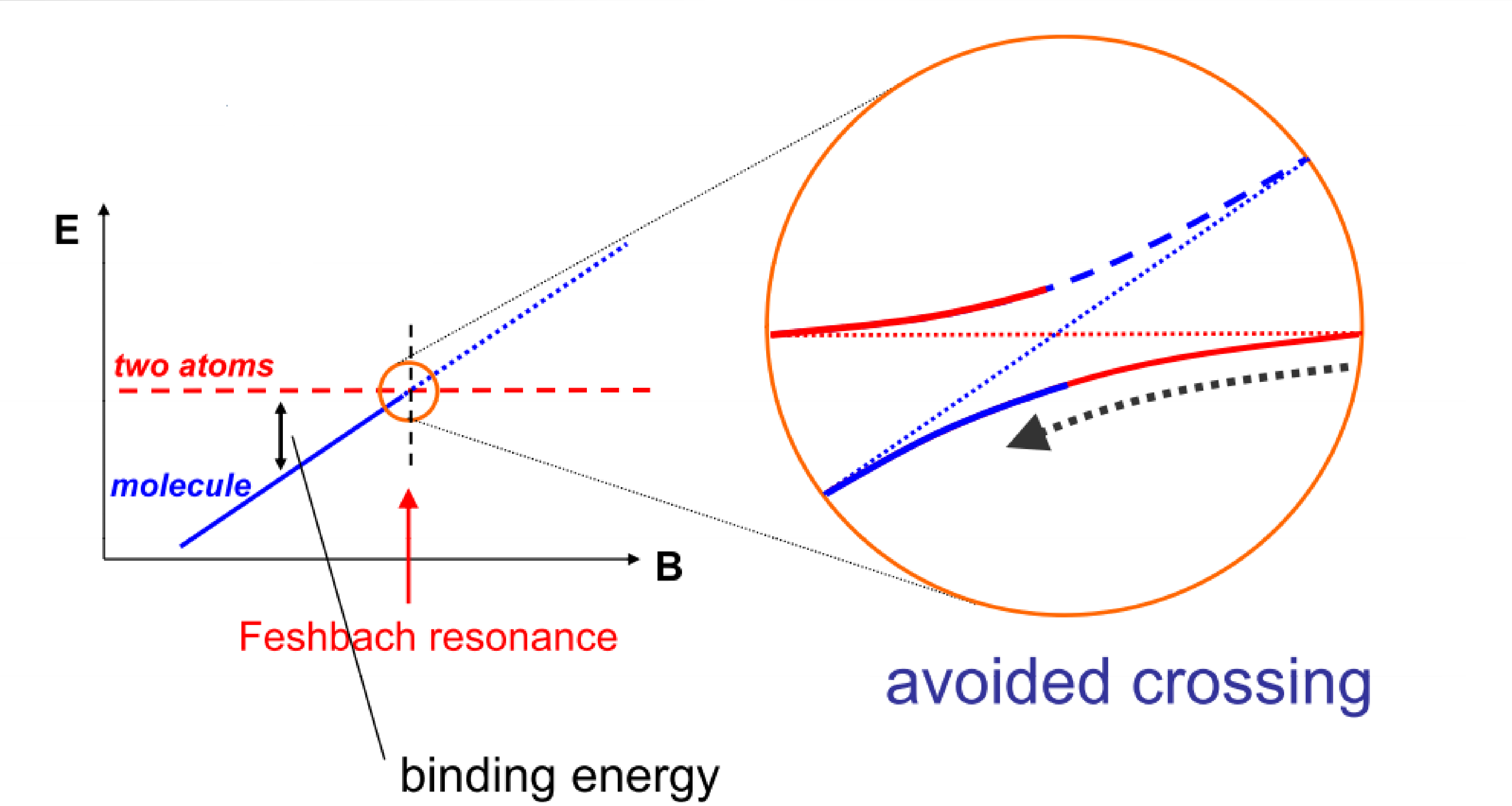
- $a(B) = a_{bg}(1 - \frac{\Delta}{B - B_0})$
- Pairing accures for $a > 0$
- Linear behavior w.r.t. magnetic detuning of chanels

$$E_B \sim \delta\mu$$

- Quatratic behavior close to the resonance

$$E_B = \frac{\hbar^2}{2ma^2}$$

Forming molecules



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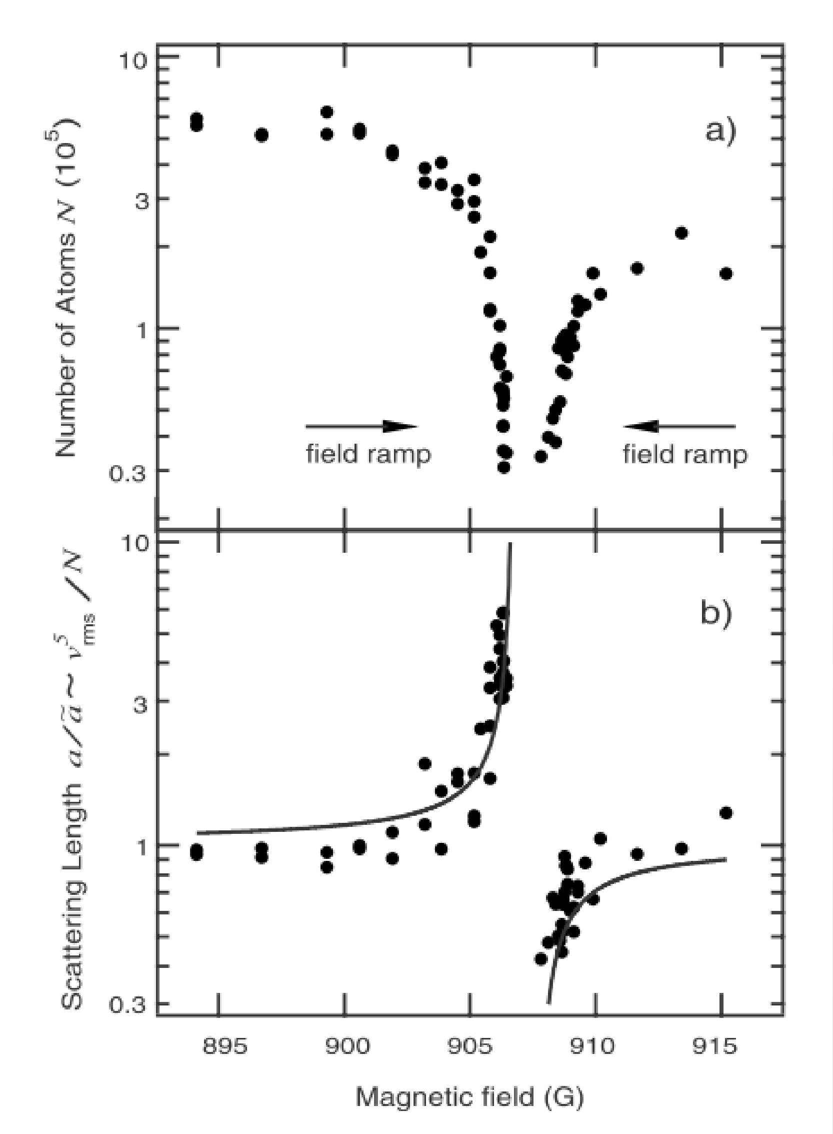
BEC of Feshbach molecules

- Atom-molecule and molecule-molecule scattering lead to big losses
- Inelastic scattering
- Close to the resonance
- Solution:
  - Use fermions



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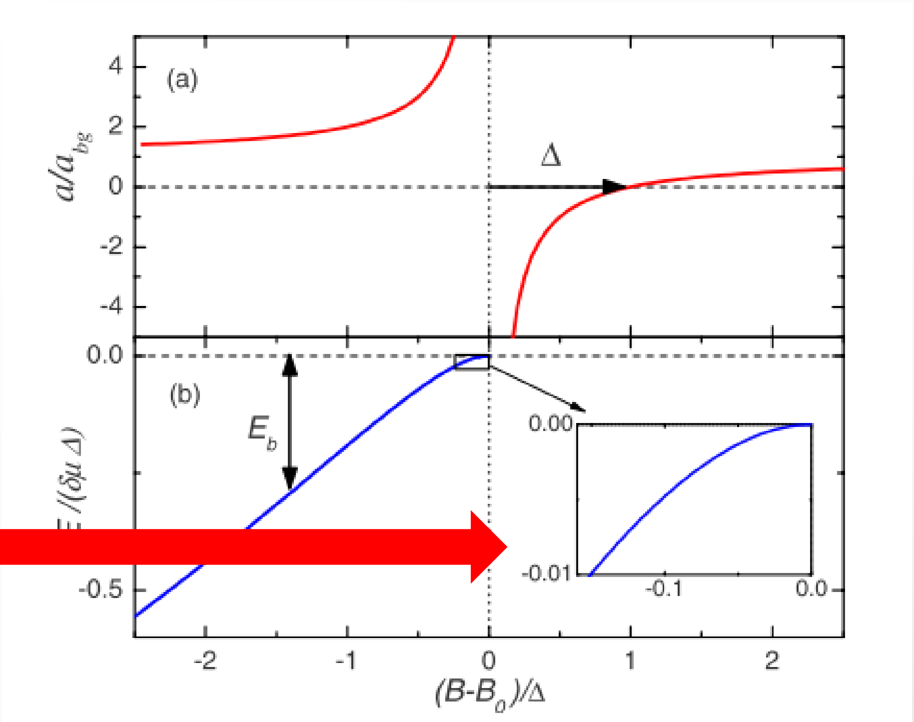
BEC of molecules

- Close to B_0 formation of halo dimers
- Dimer is characterized only by
- $E_B = \frac{\hbar^2}{2ma^2}$
- Dimers are stable against inelastic decay



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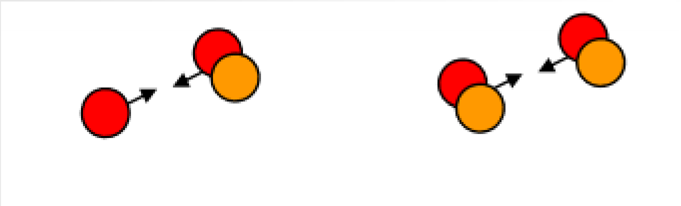
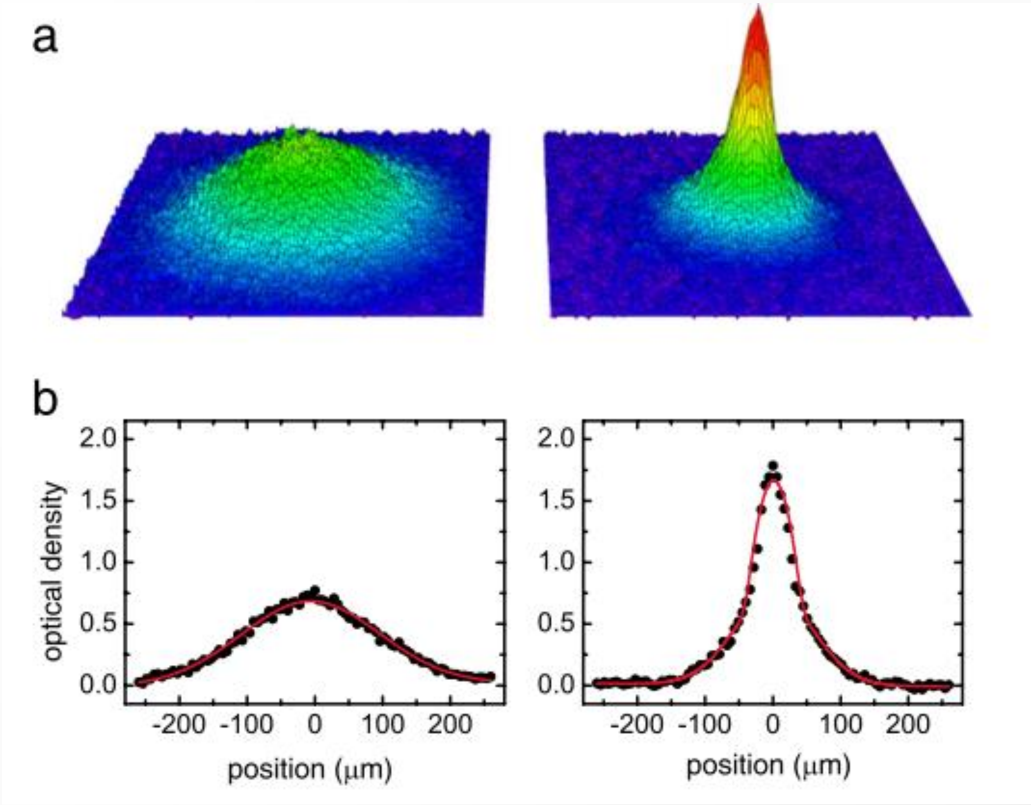
BEC of molecules

- Pauli blocking



- Inelastic scattering reduced
- BEC can be achieved



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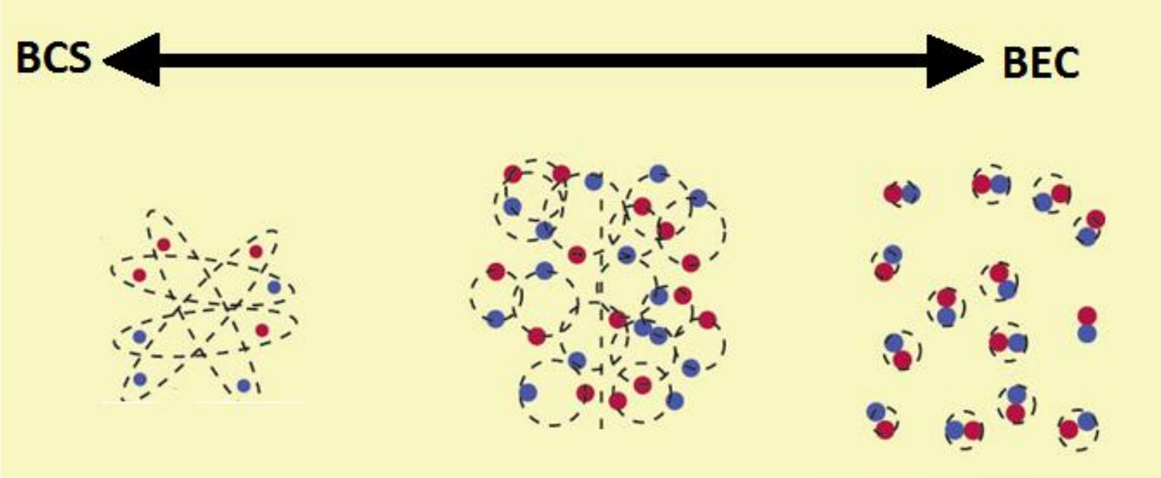
BCS-BEC-Crossover

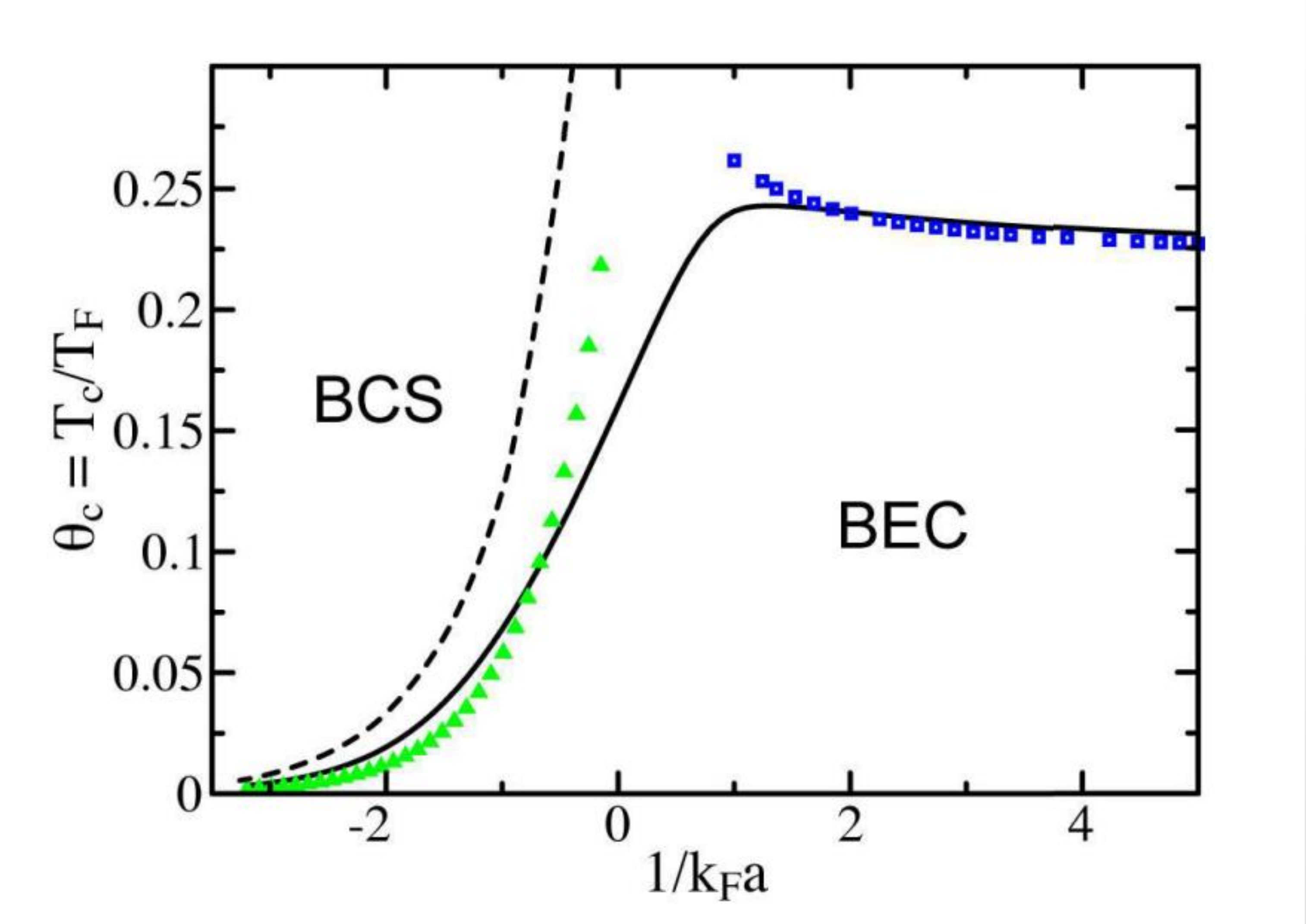
- Feshbach molecules already indicate a crossover
- Fermions → Dimers have a long lifetime near B_0
- Investigation is possible



BCS-BEC-Crossover

- BCS-Side:
  - Fermi gas → Superfluid
  - Interparticle spacing much smaller than pair size
- Intermediate regime
  - Interparticle spacing in the order of pair size
- BEC-Side
  - bosonic Molecules undergo BEC
  - Interparticle spacing much larger that pair size
- BCS-BEC-Crossover is not a phase transition!!!

BCS-BEC Crossover



Source: http://arxiv.org/pdf/0704.3011

BCS-Side

- Attractive regime or weak coupling regime

$$k_F|a| \ll 1$$

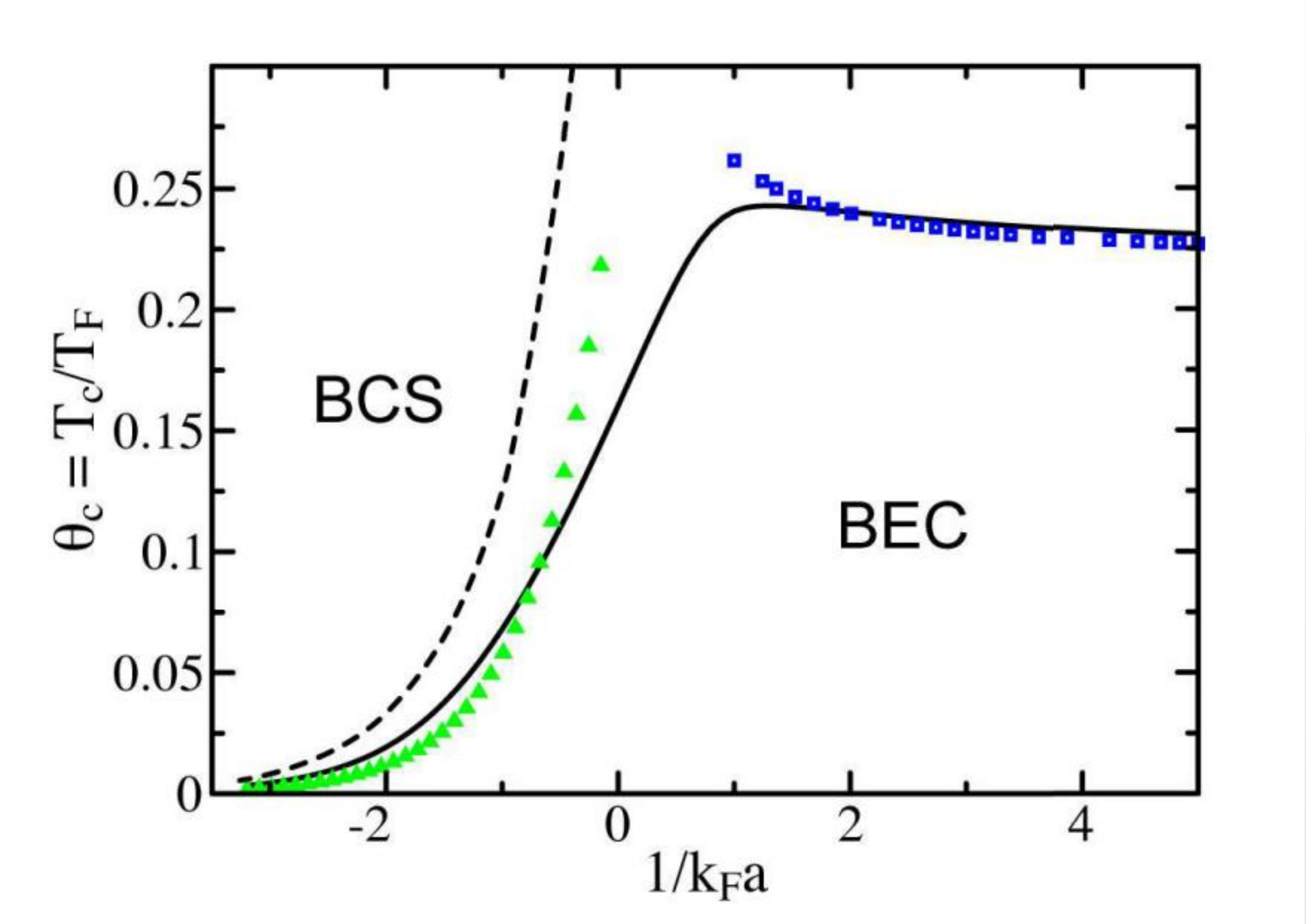
$$T_C = \frac{8e^C}{(4e)^{\frac{1}{3}}\pi e^2} T_F \exp\left(-\frac{\pi}{2k_F|a|}\right)$$

- C=0.577 and e=Eulers constant
- Describes behavior of weak coupling regime
- Limited by finite level spacing or trap size

BEC-Side

- $k_F a \ll 1$
- E_B is much larger than E_F
- If $k_B T \ll E_B$ purely bosonic describtion of a dilute gas of strongly bound pairs
- Description via Gross-Pitaevskii equation
- $T_C(a \to 0) = 0.218\, T_F$
- Independent of coupling constant

BCS-BEC Crossover



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BCS-Side/BEC-Side

- BCS:
  - Condensation to superfluidity due to gain in potential energy
  - Energyscale $\sim\Delta$
  - Dependent on a
- BEC:
  - Condensation to superfluidity due to gain in kinetic energy
  - Not dependent on a

Unitarity

- Strong coupling regime
- $k_F|a| \geq 1$
- T_C is in a accessible range of order T_F
- No analytical solution
- No perturbation theory applies
- Three and four body collisions cannot be neglected

Unitarity

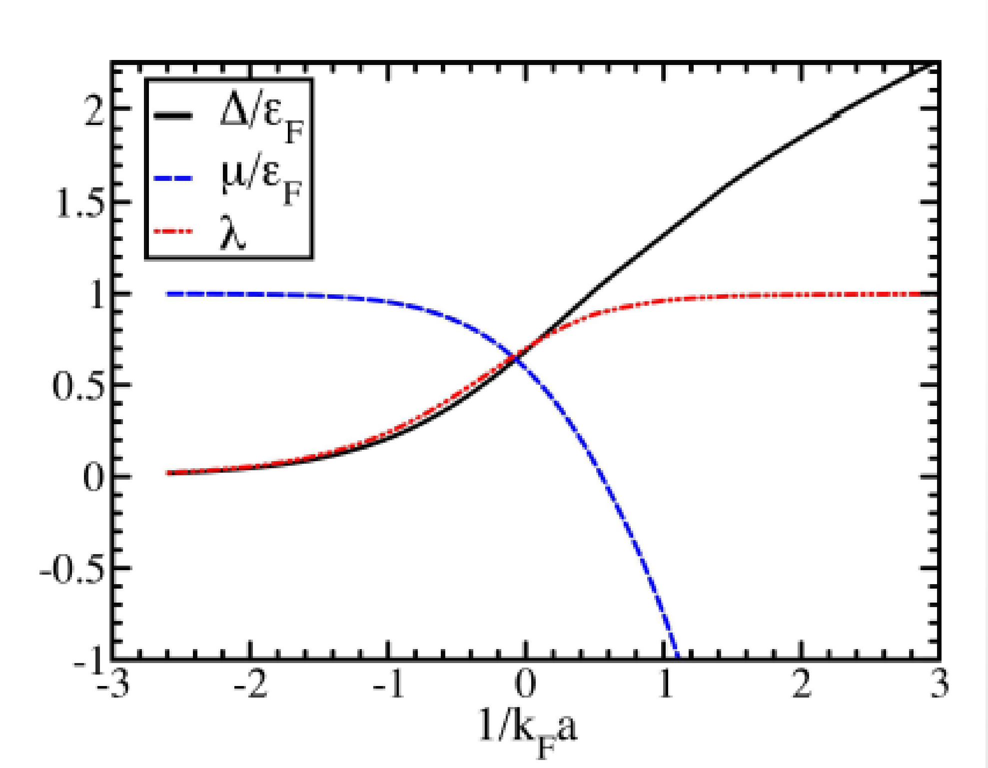
- Preformed pairs can exist above T_C without global superfluidity
- Regime can have analogies to high T_C SC
- Liquid above T_C has interesting properties
  - E.g. Supressed magnetic susceptibility

Extented BCS-Theory

- Applying BCS-Theory to the crossover
- By regularizing gap equation
- Three important parameters:
  - μ: chemical potential
  - Δ: gap energy as order parameter
  - λ: Ratio between particles that undergo BSC condensation and particles that undergo BEC condensation

Extented BCS-Theory

- Smooth transition
- No phase transition



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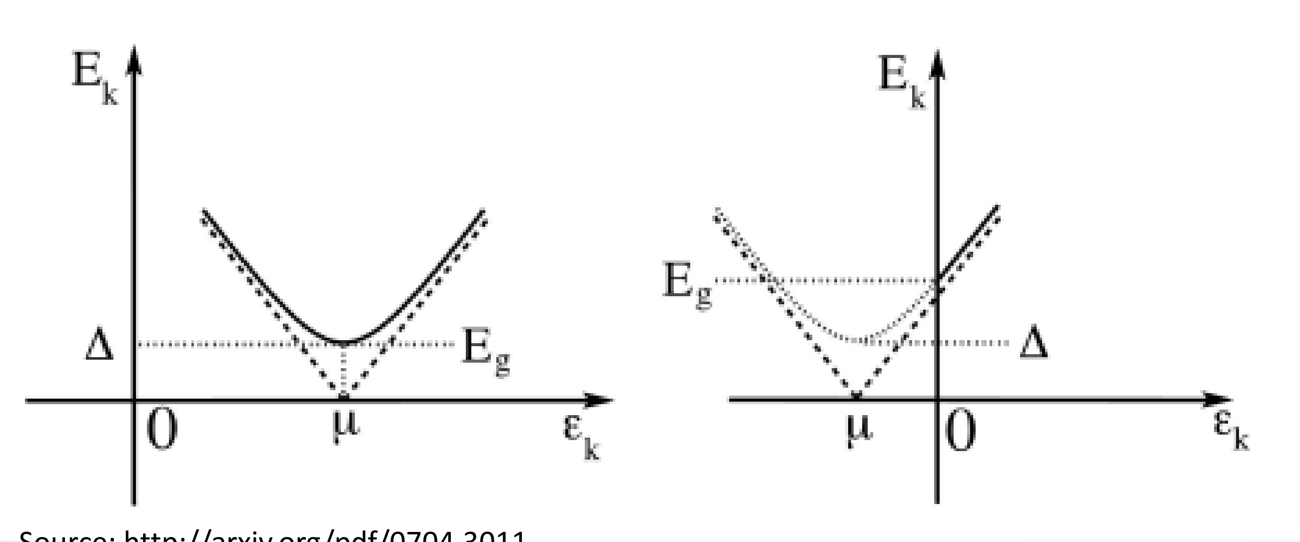
- BCS-side: $\mu \longrightarrow \varepsilon_F$ and $\Delta \sim \exp(-\frac{1}{N(0)g})$
- BEC-side: $\mu \longrightarrow -\frac{\varepsilon_B}{2}$ and Δ increasing

Extented BCS-Theory



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- 2Δ differs from ε_B of strongly bound pair
- Look at excitation Energy for single particles
- $E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$ for $\mu < 0$
- this is not at Δ but at $\sqrt{\Delta^2 + \mu^2}$

Limitations

- No density fluctuations described
- Only for zero total momentum
- Only balanced crossover

Conclusion

- Gap introduced by BCS-Theory
- Create molecules that undergo BEC
- Cold fermions offer a wide field of properties
- Crossover between BCS and BEC
- At unitarity physics become very interesting but have not completely been understood

Thank you very much for your attention!

Any questions?