BCS-BEC crossover

Hauptseminar by Robin Wanke, 18.06.2013

1 BCS-Theory

- BCS-Theory was developed by Bardeen, Cooper and Schriefer in 1957
- Two electrons with opposite momentum and spin form a bosonic pair (Cooper pair)
- Effective interaction is attractive due to phonon coupling
- Formation of a condensate with zero resistance

In second quantization the groundstate is given by

$$|\psi_G\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|\phi_0\rangle$$

$c^\dagger$: fermionic creation operators. $|v_k|^2$ ($|u_k|^2$): propability of (no) Cooper pair being formed
Reduced Hamiltonian

$$H = \sum_\sigma \sum_k \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_{kk'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

Bogoliubov like transformation to solve Hamiltonian, using quasiparticle operators

$$c_{k\uparrow} = u^*_k a_{k0} + v_k a^\dagger_{k1}$$
$$c_{-k\downarrow} = -u^*_k a_{k0} + u_k a^\dagger_{k1}$$

The gap is the order parameter of the system. For the weak coupling regime (valid for all conventional superconductors)

$$\Delta = 2\hbar\omega_D e^{-\frac{1}{\lambda N(0)}}$$

λ: coupling parameter, ω_D: Debye frequency, $N(0)$: Density of states

2 Building molecules

2.1 Building molecules

- Feshbach resonances are used to create bosonic molecules out of fermions
- Fast ramping to $a < 0$ by increasing B-field above the feshbach resonance
- Slowly turning the B-field down
- Avoided crossing leads the atoms into a bound state

2.2 BEC of molecules

A BEC with molecules has not yet been reached. But artificially made molecules can. Close to the feshbach resonance where $a > 0$, halo dimers can form that are characterized by

$$E_B = \frac{\hbar^2}{2ma^2}$$

only. Those dimers are stable against large decay that usually happens.
BEC can be reached!

3 BCS-BEC crossover

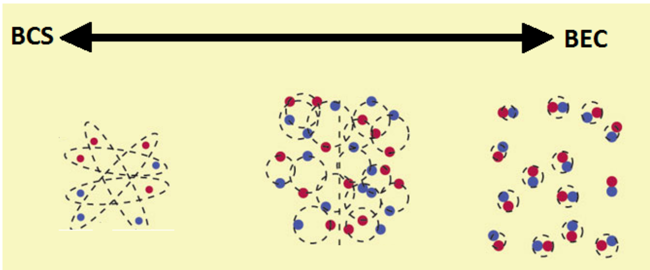

Figure 1: BCS-BEC crossover

3.1 BCS-Side

- Attractive regime ($a < 0$)
- Interparticle spacing is much smaller than molecule size
- $$T_C \approx T_F \exp\left(-\frac{\pi}{2k_F|a|}\right)$$

3.2 BEC-Side

- Repulsive regime ($a > 0$)
- Interparticle spacing is much larger than pair size
- $E_B \gg E_F$ and $k_B T \ll E_B$ purely bosonic description for a dilute gas of strongly bound pairs (Gross-Pitaevskii equation)
- $T_C(a \leftarrow 0) = 0.218 T_F$ independent of the coupling constant!

3.3 Intermediate regime

- interparticle spacing has the same size than pair size
- No analytical solution
- No pertubation theory cannot be applied, since three and four body interaction cannot be neglected anymore