Imbalanced BCS-BEC crossover

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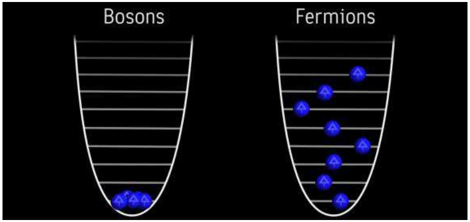
18. 06. 2013



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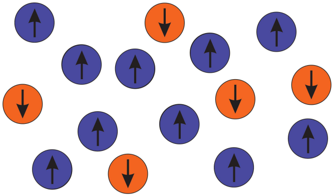
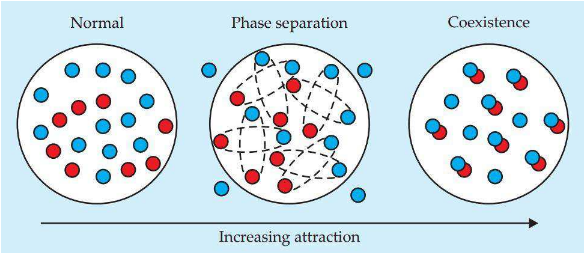




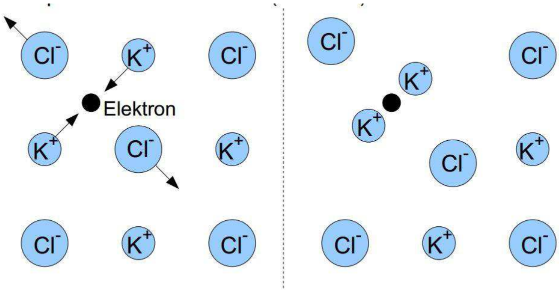
C. A. R. Sa de Melo, Physics Today, 45-50 (2008).

Outline

1. Polaron problem
2. Superfluidity for fermions in imbalanced situations
3. FFLO correlations in the partially polarized phase

Polaron problem

- Polaron is a quasi-particle composed of a charge (e^-) and its polarization field
- For example the interaction between an electron and a lattice $\rightarrow$ leads to a deformation of the lattice



Kittel, Einführung in die Festkörperphysik, Oldenbourg Wissenschaftsverlag (2005).

- We can describe this problem with a free quasi particle with effective mass m^{**} and self energy E_0
- Hamiltonian of free particle ($V = 0$ with effective mass m^*) for fermions

$$H_f = -\frac{\hbar^2}{2m^*}\Delta \tag{1}$$

- Hamiltonian of harmonic oscillator (atoms)

$$H_{ho} = \hbar\omega \sum_q b_q^\dagger b_q \tag{2}$$

- Hamiltonian of e^--phonon interaction

$$H_{WW} = \hbar \sum_q \left(g_q e^{iqx} b_q + g_q^* e^{-iqx} b_q^\dagger \right) \tag{3}$$

Fröhlich Hamiltonian

- Fröhlich Hamiltonian for polarons:

$$H_{polaron} = H_f + H_{ho} + H_{WW} = \\ -\frac{\hbar^2}{2m^*}\Delta + \hbar\omega \sum_w b_q^\dagger b_q + \hbar \sum_q \left(g_q e^{iqx} b_q + g_q^* e^{-iqx} b_q^\dagger \right) \quad (4)$$

- Blackboard calculation to derive the effective mass and the self energy of the polaron

Results of blackboard calculation

- Energy of polaron E_k:

$$E_k = -\hbar\omega\alpha + \frac{\hbar^2 k^2}{2m^*}\left(1 - \frac{\alpha}{6}\right) \tag{5}$$

- Selfenergy E_0:

$$E_0 = -\alpha\hbar\omega \tag{6}$$

$\rightarrow$ Energy offset

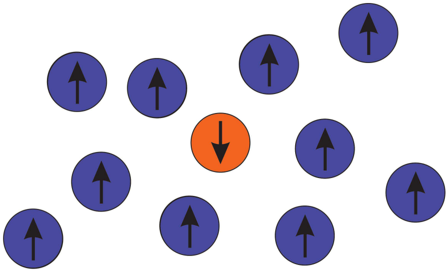
- Effective mass m^{**}

$$m^{**} = m^*\left(1 + \frac{\alpha}{6}\right) \tag{7}$$

$\rightarrow$ Heavy electron

Polaron problem

- Mobile impurity can also be a single spin-↓ fermion in a Fermi sea of spin-↑ fermions
- Polaron is limiting case of a strong imbalanced Fermi gas



What happens for more polarons

- Two component $(\uparrow, \downarrow)$ 2D model with short range interaction described by:

$$H = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}\sigma} - \mu_\sigma) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{\Omega} \sum_{\mathbf{k},\mathbf{k'},\mathbf{q}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{k'}\downarrow} c_{\mathbf{k'}+\mathbf{q}\downarrow} c_{\mathbf{k}-\mathbf{q}\uparrow} \tag{8}$$

  with
  $\epsilon_{\mathbf{k}\sigma} = \mathbf{k}^2/2m_\sigma$ the single particle energy,
  μ_σ the chemical potential,
  Ω the system area and
  g the contact interaction parameter

- No interaction between spins of same species (Pauli blocking)

- For weak interaction: density fluctuations of the Fermi sea
  $\rightarrow$ Polaron state

- Polaron ground state for weak interaction regime:

$$|P\rangle = \alpha_0 c^\dagger_{0\downarrow}|N\rangle + \frac{1}{\Omega}\sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} c^\dagger_{\mathbf{q}\text{-}\mathbf{k}\downarrow} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\uparrow} \qquad (9)$$

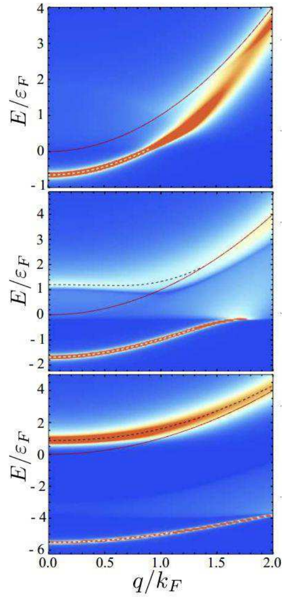
- For increasing interaction the impurity binds a $\uparrow$ fermion to build a molecule

- The polaron state separates into two branches of different polarons:
  $\rightarrow$ low-energy polarons which interacts attractively with the Fermi gas
  $\rightarrow$ a metastable excited repulsive polaron at intermediate binding energy

- With RF spectroscopy both branches are detectable

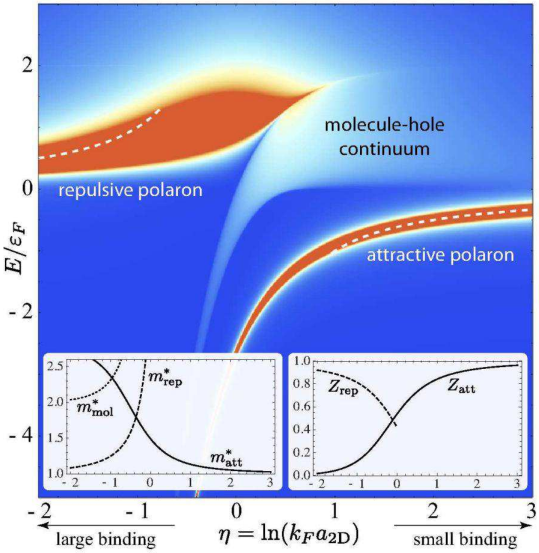
Polaron state

- Theoretical results of polaron dispersion relation for increasing binding
- For weak binding $\epsilon_B \ll \epsilon_F$ we have a well defined quasi-particle at small momenta
- For intermediate binding a new repulsive polaron state rises with higher energy due to the repulsive interaction with the Fermi sea
- For strong binding both polaron branches are well separated and repulsive polaron becomes stable quasi-particle



R. Schmidt, et al., Physical Review A, **85**, 021602 (2012).

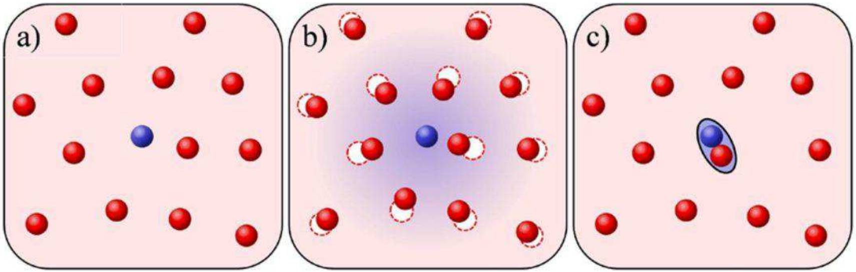
Polaron state



R. Schmidt, et al., Physical Review A, **85**, 021602 (2012).

- Properties of polaron for varying interaction parameter $\eta = \ln(k_F a_{2D})$ with the 2D scattering length $a_{2D} = \hbar/\sqrt{m\epsilon_B}$
- Continuous crossover with shifting of spectral weight Z to the repulsive polaron branch with increasing binding
- Good agreement with perturbation results (dashed lines) in both limits

Summary of polaron problem



A. Schirotzek et al., Phys. Ref. Lett. **102**, 230402 (2009).

- Depending on interaction strength we get either a polaron or molecule state

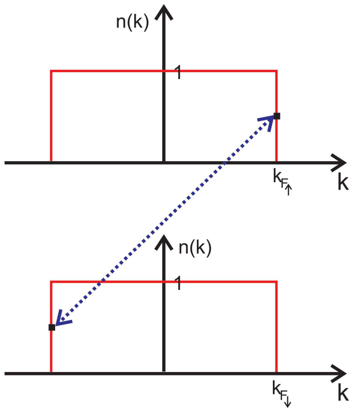
Summary of polaron problem

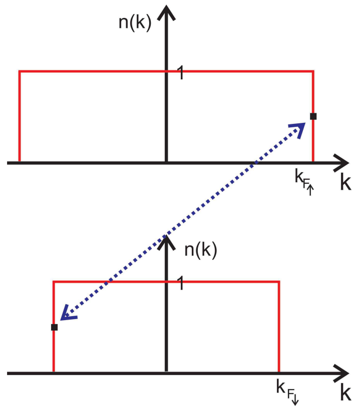
- In 3D: Polaron state for weak interaction, for increasing interaction the impurity $\downarrow$ forms a dimer (molecule) with a $\uparrow$
- In 2D the situation is less clear because of enhanced quantum fluctuations
- Possible polaron to molecule transition
- In 1D there is no transition between polaron and molecule

Spin-imbalanced Fermi gas

- We can form molecules so we want to have a look on superfluidity in these imbalanced systems
- Fermi gas with different numbers of up ($N_\uparrow$) and down ($N_\downarrow$) spins
- Explore interplay between magnetism and superfluidity
- Fulde and Ferrell and Larkin and Ovchinnikov (FFLO) proposed a pairing of fermions with finite momentum
- Normally in superconductor fermions pair to Cooper pairs (CP) which have $k = 0$ momentum

Pairing





- $|\mathbf{k}_{F_\uparrow}| = |\mathbf{k}_{F_\downarrow}|$
- Normal Cooper-pairs, $\mathbf{k}_{pair} = \mathbf{0}$

- $|\mathbf{k}_{F_\uparrow}| \neq |\mathbf{k}_{F_\downarrow}|$
- Finite momentum $\mathbf{k}_{pair} = Q \neq \mathbf{0}$

- What is the dependence of $\mathbf{k}_{pair}$ on $P = \frac{N_\uparrow - N_\downarrow}{N}$?
- Pair density of balanced gas peaks in zero momentum
- What is expected in the case of an imbalanced situation?

Exact numerical DMRG results in 1D

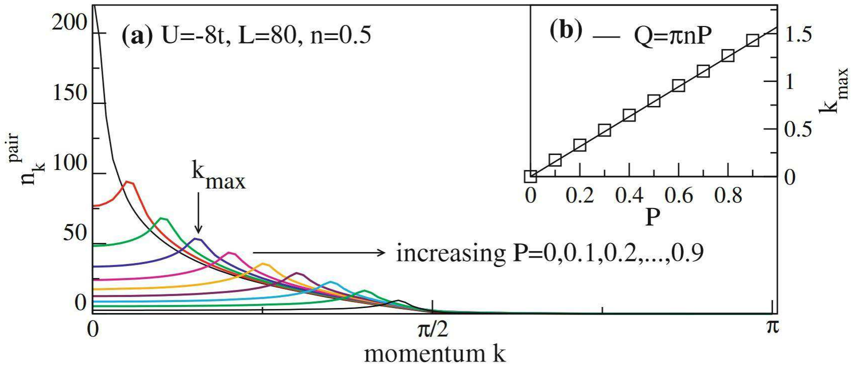
- Using the Hubbard model Hamiltonian

$$H_0 = -t \sum_{i=1\sigma}^{L-1} \left(c_{i\sigma}^{\dagger} c_{i+1\sigma} + h.c. \right) + U \sum_{i=1}^{L} n_{i\uparrow} n_{i\downarrow} \tag{10}$$

- Fourier transformation of Green functions is the momentum distribution function

$$n_k^{pair} = \frac{1}{L} \sum_{lm} \exp[ik(l-m)] \rho_{lm}^{pair} \tag{11}$$

Exact theoretical DMRG results in 1D

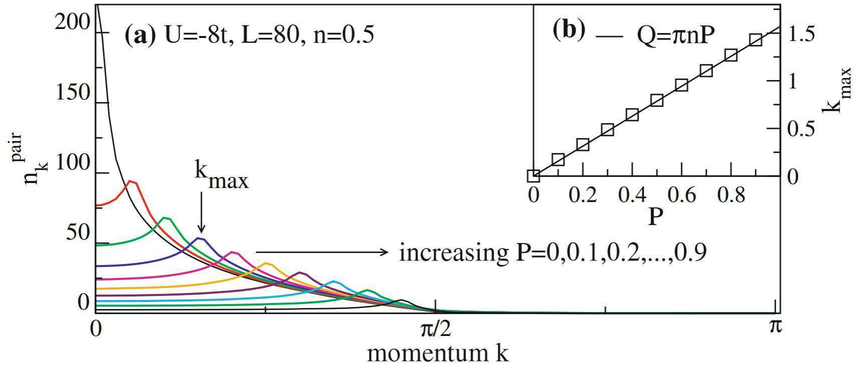


A. E. Feiguin, et al., Springer Verlag Berlin Heidelberg (2012).

- $\mathbf{k}_{pair} = Q$ has a linear dependence on P:

$$Q = k_{F_\uparrow} - k_{F_\downarrow} = \pi n P \qquad (12)$$

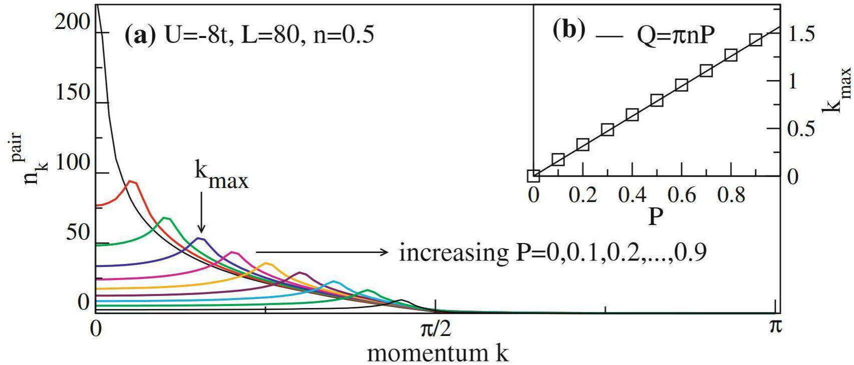
Exact theoretical DMRG results in 1D



A. E. Feiguin, et al., Springer Verlag Berlin Heidelberg (2012).

- Highest density shifts to higher momenta for increasing P

Exact theoretical DMRG results in 1D



A. E. Feiguin, et al., Springer Verlag Berlin Heidelberg (2012).

- Decreasing density for increasing P

FFLO phase

- Predicted, but has not been experimentally observed in any system
- Magnetism accommodated by pairing with finite momentum
- Magnetism is expected to have interplay with paring
  $\rightarrow$ easier to measure than pairing
- Fulde and Ferrell said that pairs form with finite momentum
- Larkin and Ovchinnikov proposed related model in which the superconducting order parameter oscillates in space
  $\rightarrow$ Can be interpreted as an interference pattern between condensates with opposite momenta

FFLO experiment

- Two spin mixture of ultracold ^{6}Li atoms trapped in 1D tubes
- Transverse confinement of fermions with trapping potential frequency $\omega_\perp$
- Strictly 1D: only lowest energy state is occupied $\epsilon_F \ll \hbar\omega_\perp$
- External magnetic field to enhance the attractive 1D interactions
- Hamiltonian (Gaudin-Yang-model):

$$H = -\frac{\hbar^2}{2m}\left(\sum_{i=1}^{N_\uparrow}\frac{\partial^2}{\partial x_j^2} + \sum_{i=1}^{N_\downarrow}\frac{\partial^2}{\partial y_j^2}\right) + g_1 \sum_{i,j=1}^{N_\uparrow,N_\downarrow} \delta\left(x_i - y_j\right) \qquad (13)$$

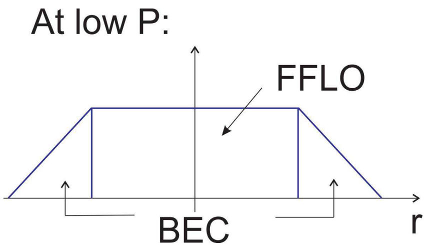
Why 1D and not 3D?

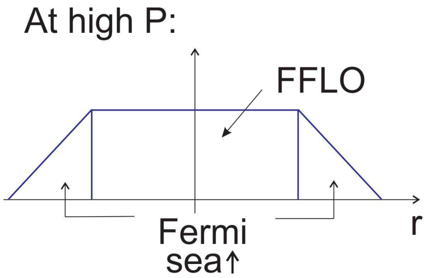
- In 1D we have an exact solution of the problem (Bethe ansatz)
- 1D Fermi gases with tunable interaction and arbitrary values of imbalance have been experimentally realized
- No FFLO phase predicted in 3D

$\rightarrow$ Hope to observe FFLO phase experimentally in 1D

Different phases in 1D

- Partially polarized core
- Surrounding wings either completely paired or fully polarized





Phase diagramm

- Chemical potential

$$\mu = \frac{1}{2}(\mu_1 + \mu_2) \quad (14)$$

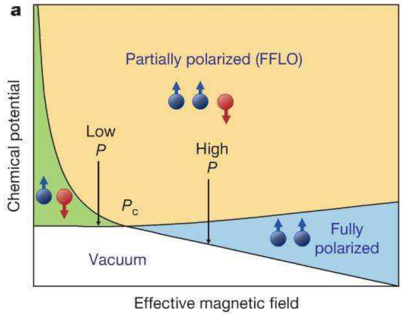
- Effective magnetic field

$$h = \frac{1}{2}(\mu_1 - \mu_2) \quad (15)$$

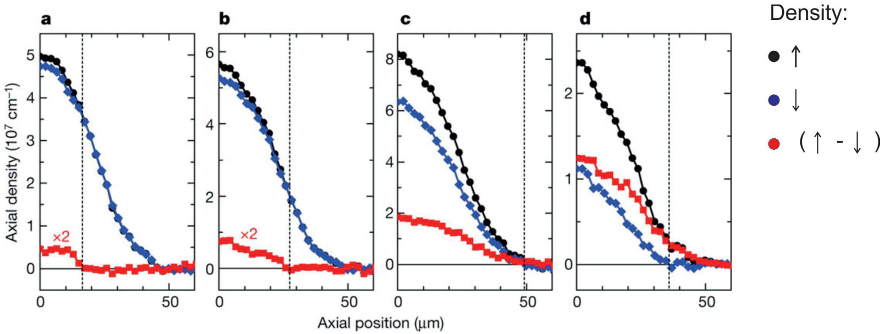
- Polarization

$$P = \frac{N_\uparrow - N_\downarrow}{N} \quad (16)$$

- μ decreases from the center to the edge of the tube, h stays constant
- Vertical arrows show FFLO phase surrounded by either fully polarized or fully paired phase



Y. Liao, et al., Nature **467**, 567-569 (2010).



Y. Liao, et al., Nature **467**, 567-569 (2010).

- Black curve shows $n_\uparrow$ of state $|1\rangle$, blue $n_\downarrow$ of state $|2\rangle$ and red $2(n_\uparrow - n_\downarrow)$
- Low P leads to PP core and fully paired wings (picture a,b)
- At $P = P_c$ we have just one phase (PP) (picture c)
- At high P we get a fully polarized wings (picture d)

- No experimental proof of these phases
- Probably fully paired phase is normal phase, because no superfluidity observed
- People are trying to find superfluidity to proof the existence of this phase

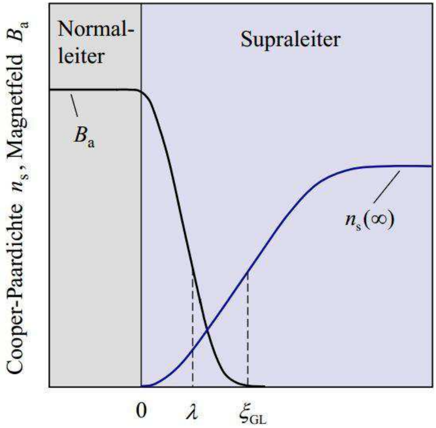
Correlation function

- Superfluid order of the ground state follows from the pair correlation

$$\rho_{ij}^{pair} = \langle c_{i,\uparrow}^{\dagger} c_{i,\downarrow}^{\dagger} c_{j,\uparrow} c_{j,\downarrow} \rangle \quad (17)$$

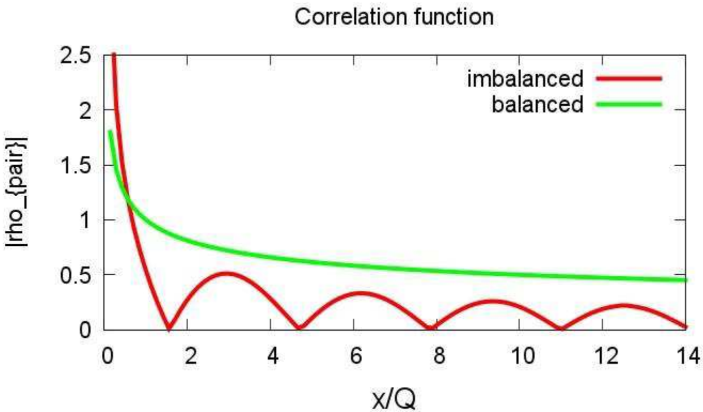
- Describes the tendency for singlet pairing as a function of separation $x = i - j$

- Correlation function of normal BCS theory decays monotonically



Kittel, Einführung in die Festkörperphysik, Oldenbourg Wissenschaftsverlag (2005).

Predictions from bosonization



- Imbalanced situation: oscillating superfluid correlation function:

$$|\rho_{ij}^{pair}| \sim |\cos(Qx)]|/x^{\Delta(p)} \qquad (18)$$

- Balanced situation: correlation exponent Δ_{SS} for singlet pairing defined by:

$$\rho_{ij}^{pair} \sim |i-j|^{-\Delta_{SS}} \qquad (19)$$

Predictions from bosonization

- For attractive interaction in balanced gases we get $\Delta_{SS} < 1$
  $\rightarrow$ decays very slowly
- Imbalanced: correlation exponent of $\Delta_{SS}(p) = \Delta_{SS}(p = 0) + 1/2$
- Thus the correlation function decays faster for imbalanced gases
- In 1D the long-range order is destroyed by quantum fluctuations
- Shown by numerical investigations (Hubbard model) using DMRG or QMC methods

Summary FFLO phase

- CP with finite momentum
- Superfluidiy is oscillating in space
  $\rightarrow$ different than in normal BCS
- Mean field theory predicts: Superfluidity low $\leftrightarrow$ high magnetism
  $\rightarrow$ It is believed that also magnetism is oscillating
  $\rightarrow$ Theoretical results are not proofed yet!

Problems of FFLO phase detection

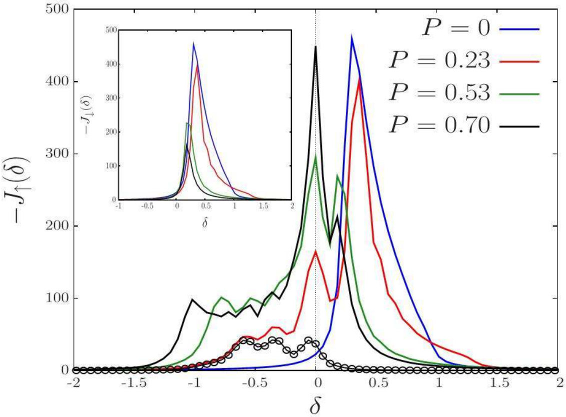
- It is difficult to cool down fermions because of interaction
- 1D it is even harder, enhanced quantum fluctuations
- Orbital pair breaking must be suppressed in superconductor $\rightarrow$ Use of organic or heavy-fermion superconductors

Proposals for the experimental observation of FFLO phase

- Unlike normal BCS, FFLO state allows single particle excitations even at $T = 0$ corresponding to unpaired particles
- These excitations produces distinct features in the RF spectra at negative energies
- Provide a signature of oscillations of the order parameter that is distinguishable from the usual pairing signatures at positive energies
- In RF spectroscopy, the internal state of one of the components ($\uparrow$ or $\downarrow$, energy $\omega_{\updownarrow}$) is coupled to a third internal state (final state, energy ω_f) by an RF pulse.

- Number of particles transferred from the initial to the final state can be observed and gives the spectrum $J_{\updownarrow}(\delta)$ as a function of the detuning δ

$$\delta = \omega_{rf} - (\omega_f - \omega_{\updownarrow}) \tag{20}$$



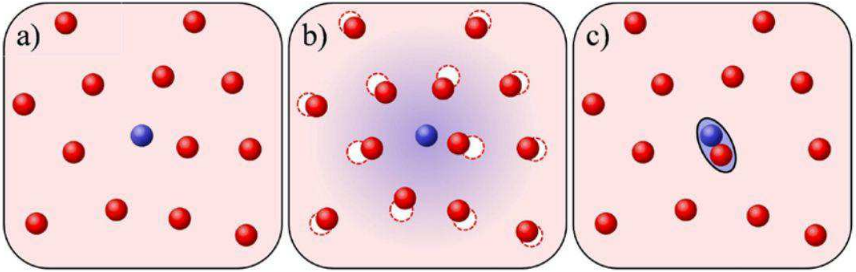
M. Reza Bakhtiari et al., Phys. Ref. Lett. **101**, 120404 (2008).

Proposals for the experimental observation of FFLO phase

- Excite spin-dipole modes with time dependent potentials and measure the response
- If oscillation are in resonance with excitation a dramatic response would indicate FFLO correlation
- One could make time-of-flight measurement to observe the velocity distribution of the condensate
- Characteristic signature of pairs with finite momenta: maximum of $n_{pair}(k)$ is at $k \neq 0$

Conclusion

- Polaron results: heavier particle, because of interaction with lattice
- Description of spin down particle in Fermi sea of spin up particles as quasi particle
- Polaron to molecule transition possible



A. Schirotzek et al., Phys. Ref. Lett. **102**, 230402 (2009).

Conclusion

- In imbalanced situation we get different phases depending on the polarization of the gas
  $\rightarrow$ Partially polarized phase
  $\rightarrow$ Fully polarized phase
  $\rightarrow$ Completely paired phase
- 1D systems necessary
- FFLO signature: $k_{pair} \neq 0$
- FFLO phase predicted but experimentally yet not observed, but there are promising suggestions