Rydberg Atoms

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06/25/2013,
Hauptseminarvortrag

Outline

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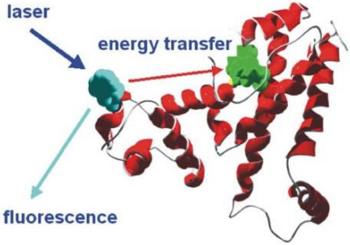
Several applications of Rydberg atoms

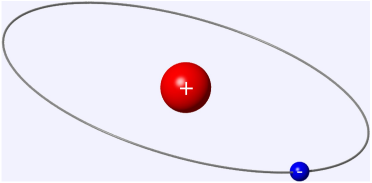
- Quantum computation: 1- and 2-Qubit operations
- Photon detection due to high sensitivity to extern fields
- Quantum simulations
- Biological issues: Foerster-resonance



Outline

Definition

- Rydberg atom has at least 1 electron far away from its core (compared to ground state electrons)
- High principle quantum number n, typically $n \geq 20$
- Correspondence principle applicable, which leads to Bohr model (H-atom-like)
- Rydberg atoms show exaggerated properties



Estimated Properties - Scaling with n

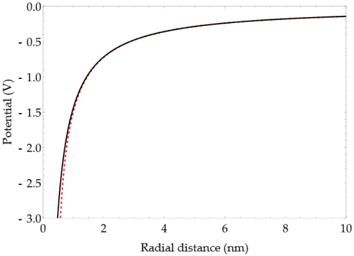
Size	$r = a_0 \cdot n^2 \sim n^2$
Energy	$E = \frac{-E_R}{n^2} \sim n^{-2}$
Level spacing	$\Delta E \sim n^{-3}$
Geom. cross section	$\sigma = \pi r^2 \sim n^4$
Dipole moment	$d = e \cdot r \sim n^2$
Polarizability	$\alpha = \frac{d^2}{\Delta E} \sim n^7$
vdW parameter	$C_6 = \frac{-(d_1 \cdot d_2)^2}{\Delta E} \sim n^{11}$
Radiative lifetime	$\sim n^3$

Quantum defect

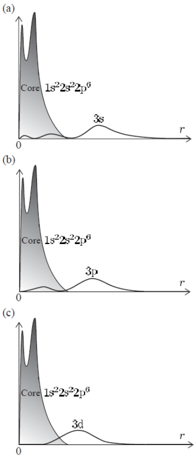
Approximation valid, if

- High-n-electron sees attractive Coulomb-potential with 1 elemental charge
- Inner shell electrons screen core

High distances (high n), high angular momenta → low spatial overlap between electron wavefunctions → Alkali atoms



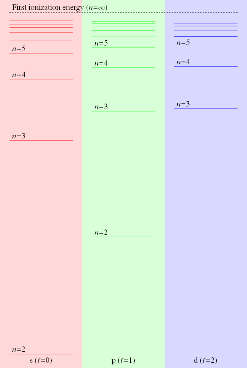
Quantum defect



Quantum defect

Effects of low angular momenta and high spatial overlap respectively:

- Penetration of inner shells leads to higher charges and greater forces
- Passing electron polarizes ion-core: Induced dipole-charge-interaction
- l-degeneracy is lifted



Empirical energy equation:

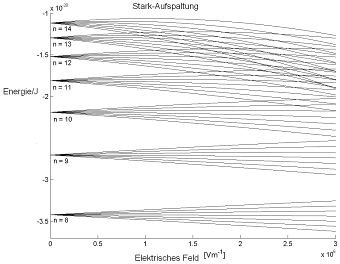
$$E_B = -\frac{E_{R,2}}{(n-\delta_{n,l})^2} \tag{1}$$

Outline

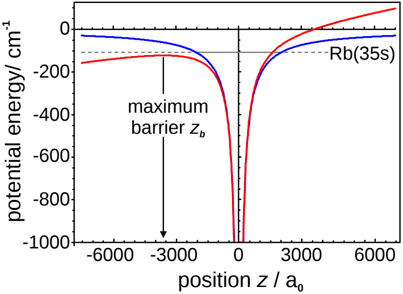
Stark effect and ionisation

Application of a static, external electric field shows

- Linear Stark effect for degenerate states (high l) $\Delta E = d \cdot E$
- Quadratic Stark effect for non-degenerate states (low l) $\Delta E = -\frac{1}{2} |\alpha| E^2$
- Ionisation at high electric fields



Ionisation



$$|E| = \frac{\pi\varepsilon_0 E_R^2}{e^3}\frac{1}{n^4} \qquad (2)$$

Atom-Light Interaction

Assuming a 2-level atom with dipole approximation:

$$\hat{H} = \hat{H}_{atom} + \hat{H}_{light} + \hat{H}_{interaction} \tag{3}$$

$$\hat{H}_a = \frac{\hat{p}^2}{2m} + \hbar\omega_0 \left|e\right\rangle \left\langle e\right| \tag{4}$$

$$\hat{H}_L = \hbar\omega_L(\hat{a}^\dagger \hat{a} + \frac{1}{2}) \tag{5}$$

$$\hat{H}_i = -d \cdot E(r, t) \tag{6}$$

- Ultracold, frozen atom $\rightarrow$ neglect kinetic energy
- Semiclassical picture $\rightarrow$ neglect quantisation of light field

Atom-Light Interaction

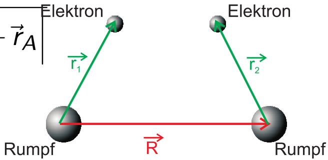
Atom-light interaction of a single atom:

$$\hat{H} = -\frac{\hbar\Delta}{2}(1-\sigma_z) + \hbar\Omega\sigma_x \tag{7}$$

Dipole-dipole interaction

Consider two H-atoms in
Born-Oppenheimer approximation

$$\hat{H}_{int} = \frac{e^2}{R} + \frac{e^2}{\left|\vec{R}+\vec{r}_B-\vec{r}_A\right|} - \frac{e^2}{\left|\vec{R}+\vec{r}_B\right|} - \frac{e^2}{\left|\vec{R}-\vec{r}_A\right|} \tag{8}$$



Expanded for $R \gg a_0$ in $\frac{r_i}{R}$ gives

$$\hat{H}_{int} = \frac{1}{R^3}[\vec{d}_A\vec{d}_B - \frac{3(\vec{d}_A\cdot\vec{R})(\vec{d}_B\cdot\vec{R})}{R^2}] = \frac{C_3}{r^3} \tag{9}$$

- Strong, long range interaction due to $d \sim n^2$
- Permanent dipoles for $l \geq 4$, for lower l only in external fields

van der Waals interaction

Perturbation theory with dipole Hamiltonian:

$$E_1 = \left\langle \psi_0 \middle| \hat{H}_{int} \middle| \psi_0 \right\rangle = 0 \tag{10}$$

Assuming a two level atom with $\langle e | d_i | g \rangle = \frac{d}{\sqrt{3}}$

$$E_2 = \frac{\left(\left|\left\langle g \middle| \hat{H}_{int} \middle| e \right\rangle\right|\right)^2}{E_g - E_e} = \frac{4d^4}{9} \frac{1}{E_g - E_e} \frac{1}{R^6} = -\frac{C_6}{r^6} \tag{11}$$

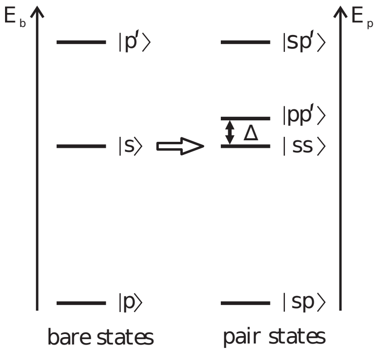
Pair states

Consider two Rydberg atoms in state $|s\rangle$, energy difference gives system $|ss\rangle$, $|p'p\rangle$ with $\Delta = (E_s - E_p) - (E_{p'} - E_s)$. Then $\hat{H} =$

$$\begin{pmatrix} 0 & U(r) \\ U(r) & \Delta \end{pmatrix}$$

with $U(r) = \frac{\vec{d}_1 \cdot \vec{d}_2}{r^3}$ gives the new eigenenergies

$$E_\pm = \frac{\Delta}{2} \pm \sqrt{(\frac{\Delta}{2})^2 + U(r)^2} \quad (12)$$



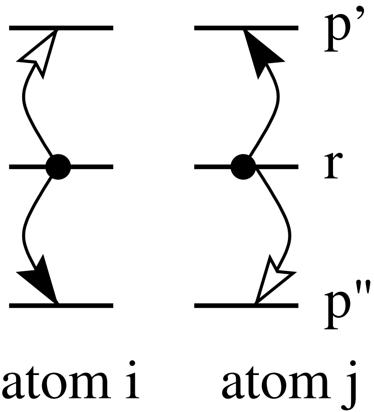
Pair states

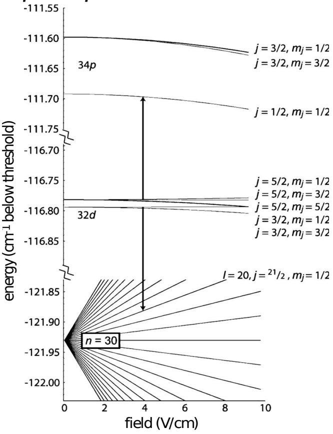
$$E_{\pm} = \frac{\Delta}{2} \pm \sqrt{(\frac{\Delta}{2})^2 + U(r)^2} \tag{13}$$

Case 1: $U(r) \gg \Delta$ Dipole interaction: $E_{\pm} = \frac{C_3}{r^3}$
$\Delta = 0 \rightarrow$ *Förster-resonance*
Case 2: $\Delta \gg U(r)$ Taylor expansion leads to: $E_{\pm} = \Delta \pm \frac{U(r)^2}{\Delta}$
van der Waals interaction $\rightarrow$ *Rydberg blockade*
Case 3: $U(r) \approx \Delta \rightarrow$ *Crossover*

Förster resonance

Non-radiative coupling $ns + ns \longleftrightarrow np + np'$

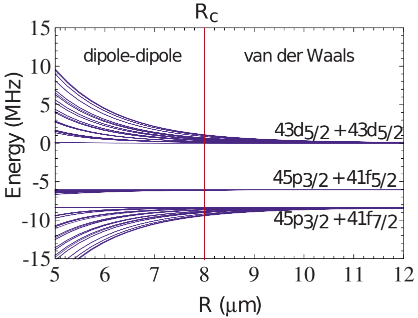




Crossover

Since there's a transition from van der Waals to dipole-dipole dominated region at $U(r) \approx \Delta$, one can define a crossover distance

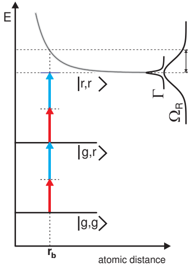
$$\frac{C_3}{r_C^3} = \Delta \tag{14}$$

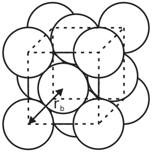


Rydberg blockade

- spatial dependant energy shift
- compare to line (saturation) broadening
- no other Rydberg atom in blockade sphere $\rightarrow$ sort of lattice order
- strong interaction $r_B \gg r_{particle}$, typically $\frac{10^3 particles}{sphere}$

$$Z\frac{C_6}{r_B^6} = ZC_6 n_r^2 = \sqrt{N}\hbar\Omega_R \qquad (15)$$





Collectivity

Laser acts on bunch of atoms:

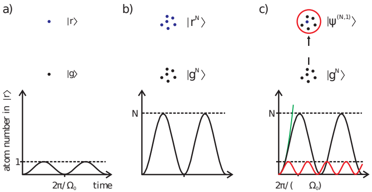
$|0\rangle = |g_1, g_2, ...g_N\rangle$

Superatom:

$|1\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |g_1, g_2, ..., r, ..., g_N\rangle$

One atom excitation:

$\hbar\Omega_0 = \left\langle g \middle| \hat{H}_{Laser} \middle| r \right\rangle$



$$\hbar\Omega_C = \left\langle 0 \middle| \hat{H}_{Laser} \middle| 1 \right\rangle = \hbar\sqrt{N}\Omega_0$$

Using $N(r) = \frac{n_g(r)}{n_R(r)}$ and blockade radius:

$$n_R(r) = \left(\frac{2\hbar}{ZC_6}\right)^{\frac{2}{5}} (n_g(r)\Omega_0^2)^{\frac{1}{5}}$$

$$\Omega_N(r) = \left(\frac{ZC_6}{2\hbar}\right)^{\frac{1}{5}} (n_g(r)\Omega_0^2)^{\frac{2}{5}}$$

Outline

1. Motivation
2. Definition and characteristics
3. Interactions
4. Experimental realisation
5. Universal scaling theory
6. Rydberg Phase Gate

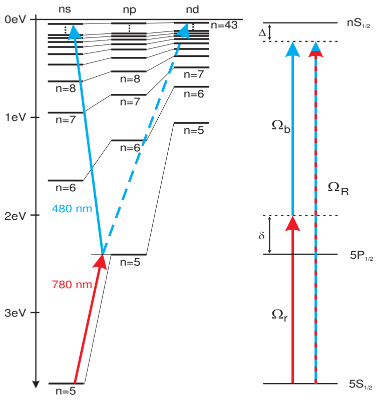
Method

Requirements

- Sufficiently intense and narrow laser for coherent evolution within natural lifetime. Intensity to overcome low dipole matrix elements and coherence for full Rabi oscillations
- Detection via field ionisation $\rightarrow$ fast ion detection (most common)
- Low temperatures to fulfill frozen gas approximation, low black body radiation, no collisions, low Doppler shift (counterpropagating lasers)
- Excitation from ultracold gas or BEC

Experiment

- Selection rules: Conservation of angular momentum
- Effective 2-level system due to large detuning δ; 5p not populated
- $\Omega_R = \sqrt{\frac{\Omega_r^2 \Omega_b^2}{4\delta^2} + \Delta^2}$



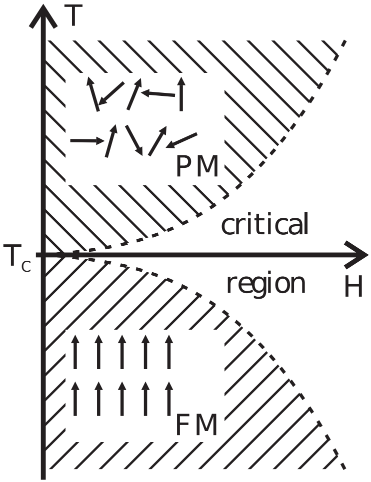
Outline

Universality

- tool to describe system without knowing microscopic details
- macroscopic behaviour is dominated by long-range physics
- critical phenomenom, f.e. near critical point of 2nd order phase transition
- only one characteristic length scale ξ
- critical exponents are universal

Example: Ferromagnet

- $T > T_C$ demagnetized, rotational symmetry
- $T_C > T$ symmetry lose, order parameter $\vec{M}$
- conjugate field H, reduced temperature $t = \frac{T}{T_C} - 1$, diverging length scale $\xi \sim \frac{1}{t^\nu}$
- $H = 0$: $M \sim t^\beta$; $t = 0$: $M \sim H^{\frac{1}{\delta}}$
- powers of H, t: universal critical exponents

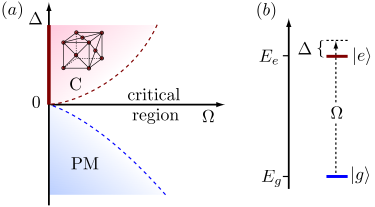


Rydberg atoms

$$\hat{H} = -\frac{\hbar\Delta}{2}\sum_i (1-\sigma_z^{(i)}) + \frac{\hbar\Omega}{2}\sum_i \sigma_x^{(i)} + C_6 \sum_{j<i} \frac{P_{rr}^{(i)} P_{rr}^{(j)}}{|r_i - r_j|^6} \tag{16}$$

with $P_{rr}^{(i)} = |r_i\rangle\langle r_i| = \frac{1-\sigma_z^{(i)}}{2}$

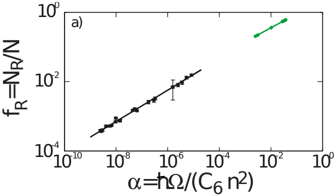
- order parameter: Rydberg fraction $f = \left\langle P_{rr}^{(i)} \right\rangle = \frac{N_R}{N_g}$
- diverging length scale $\xi = \frac{a_R}{a_g} = \frac{1}{a_g}\sqrt{\frac{C_6}{\hbar\Omega_N}}$
- 'conjugate field' $\alpha = \frac{\hbar\Omega_0}{C_6 n_g^2} = \frac{couplingstrength}{interaction}$
- $t \widehat{=} \Delta' = \frac{\hbar\Delta}{C_6 n_g^2}$



Rydberg atoms

Second order phase transition for $1 \gg \alpha$ between pure ground state (paramagnetic) and arranged Rydberg atoms in crystal (ferromagnetic)

- ξ diverges for $\alpha \to 0$
- $\Delta' = 0$: system independant of microscopic details
- $\alpha = 0$: $f \sim \Delta'^{\kappa}$; $\Delta' = 0$: $f \sim \alpha^{\nu}$
- κ, ν universal scaling exponents in critical region, i.e. $1 \gg \alpha, 1 \gg \Delta'$



Outline

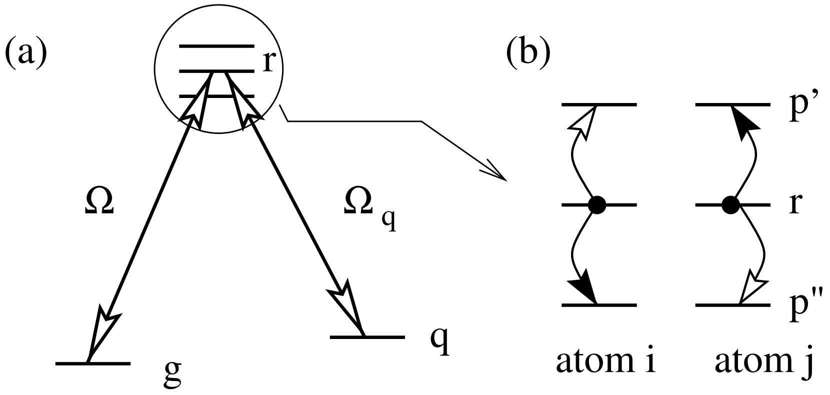
Quantum Computation and Rydberg atoms

- quantum computation requires coherent manipulation of large number of coupled quantum systems
- storage of information in collective excitations of many-atom ensembles
- single atom absorption cross-section generally low

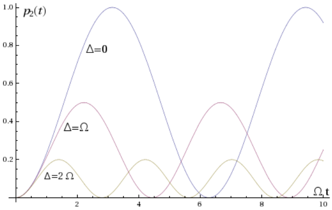
$\rightarrow$ make use of

- collective behaviour of Rydberg excitations
- Rydberg blockade control and strong interaction
- well developped techniques (cooling, trapping, ...)

Rydberg storage and switch



Two-level dynamics

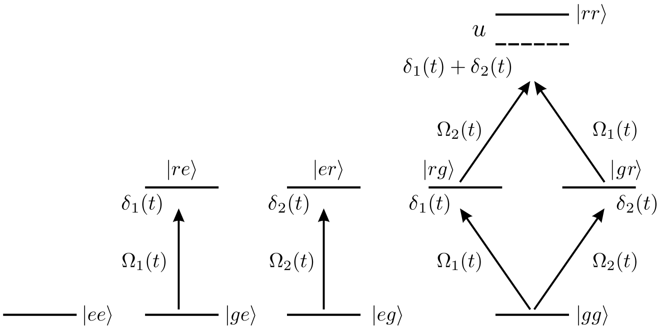


$$\begin{pmatrix} |g(t)\rangle \\ |r(t)\rangle \end{pmatrix} = \begin{pmatrix} cos(\theta(t)) & -i \cdot sin(\theta(t)) \\ i \cdot sin(\theta(t)) & cos(\theta(t)) \end{pmatrix} \begin{pmatrix} |g(0)\rangle \\ |r(0)\rangle \end{pmatrix}$$

with $\theta(t) = \sqrt{N} \int_0^t \frac{\Omega(\tau)}{2} \mathrm{d}\tau$

- Population transfer via π-pulse: $\pi = \int \Omega_f(\tau) \mathrm{d}\tau$
- Arbitrary manipulation from known state to final state possible

Phase Gate



Realisation of a phase gate (total phase shift of π):

- $|g,e\rangle \rightarrow^{\pi} |r,e\rangle \rightarrow^{\Delta t=\frac{\phi}{U(r)}} |r,e\rangle \rightarrow^{\pi} |e,g\rangle$
- or $|g,g\rangle \rightarrow^{\pi} |r,g\rangle \rightarrow^{2\pi} (-1)\cdot|r,g\rangle \rightarrow^{\pi} |g,g\rangle$
- Error sources: Dephasing, double excitation

Conclusion

- Rydberg atom H-like behaviour (Bohr atom, quantum defect) and extraordinary properties
- Interactions in picture of two level atom
- Experiment
- Quantum critical behaviour, universality and phase diagram
- Examples of quantum computation with Rydberg atoms