Rydberg atoms

1 Definition and characteristics

Definition

- Rydberg atoms: At least 1 electron with high principle quantum number
- This electron is far away from the core compared to the ground state atoms
- Correspondence principle $\Rightarrow$ Bohr’s model: Estimation of properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>$r = a_0 \cdot n^2 \sim n^2$</td>
</tr>
<tr>
<td>Energy</td>
<td>$E = -\frac{E_R}{n^2} \sim n^{-2}$</td>
</tr>
<tr>
<td>Level spacing</td>
<td>$\Delta E \sim n^{-3}$</td>
</tr>
<tr>
<td>Geom. cross section</td>
<td>$\sigma = \pi r^2 \sim n^4$</td>
</tr>
<tr>
<td>Dipole moment</td>
<td>$d = e \cdot r \sim n^2$</td>
</tr>
<tr>
<td>Polarizability</td>
<td>$\alpha = \frac{d^2}{\Delta E} \sim n^7$</td>
</tr>
<tr>
<td>vdW parameter</td>
<td>$C_6 = \frac{-(d_1 \cdot d_2)^2}{\Delta E} \sim n^{11}$</td>
</tr>
<tr>
<td>Radiative lifetime</td>
<td></td>
</tr>
</tbody>
</table>

Limits of model:

- Low spatial overlap between core and outer electron required for agreement with model
- Good approximation for high principle quantum number, high angular momenta
- Description by empirical formula:

$$E_B = -\frac{E_{R,2}}{(n - \delta_{n,l})^2}$$

2 Interactions

- High sensitivity to electric fields (large dipole moments): Stark effect, ionisation
- Atom light interaction using a two-level atom with a semiclassical model leads to Hamiltonian:

$$H = -\frac{\hbar}{2}(1 - \sigma_z) + \hbar \Omega \sigma_x$$

with $\sigma$ being the Pauli matrices

- Interaction between Rydberg atoms via dipole dipole interaction $\sim \frac{C_6}{r^6}$ and van der Waals interaction $\sim \frac{C_6}{r^6}$

- Collectivity effects lead to factor $\sqrt{N}$ in Rabi excitation

Treatment in pair state model leads to different regimes:

- Dipole dipole interactions for low energy level differences, even Förster resonances for vanishing energy difference
- Crossover between dipole dipole and van der Waals interactions for $U(r) \approx \Delta$
- Van der Waals interactions for high energy differences $\Rightarrow$ Rydberg blockade

Rydberg blockade:

- Compare spatial dependant energy shift with line broadening effects
- Inside Rydberg blockade sphere, second Rydberg excitation is shifted out of resonance
- Blockade radius: Biggest radius in which only one excitation is possible (at fixed laser frequencies)
• Evaluation: Comparing interaction strength and laser excitation given by Rabi frequency

\[
Z \frac{C_0}{r_B^6} = Z C_0 n_r^2 = \sqrt{N} \hbar \Omega_R
\]  

(3)

3 Universality

• Concept of universality can also be applied to Rydberg atoms

• Order parameter \( f = \left< P^{(i)}_{rr} \right> = \frac{N}{M^2} \), diverging critical length scale \( \xi = \frac{a_n}{a_g} = \frac{1}{a_g} \sqrt{\frac{C_0}{M N}} \)

• Parameters: \( \alpha = \frac{\hbar \Omega_0}{C_0 n_g^2} \) coupling strength interaction, \( \Delta' = \frac{\hbar \Delta}{C_0 n_g^2} \)

• Phase transition from no Rydberg excitation (paramagnetic) to crystal given by blockade spheres (ferromagnetic)

Total Hamiltonian:

\[
\hat{H} = -\frac{\hbar \Delta}{2} \sum_i (1 - \sigma_z^{(i)}) + \frac{\hbar \Omega}{2} \sum_i \sigma_x^{(i)} + C_0 \sum_{j<i} \frac{P^{(i)}_{rr} P^{(j)}_{rr}}{|r_i - r_j|^6}
\]  

(4)

with \( P^{(i)}_{rr} = |r_i \rangle \langle r_i| = \frac{1-\sigma_z^{(i)}}{2} \)

4 Phase gate

• Rydberg atoms useful for quantum computation because of collectivity behaviour and control with Rydberg blockade

• Usage of \( \pi \)-pulses in combination with Rydberg blockade effect (different states involved) offers realisation of phase gate, e.g. a phase shift of \( \pi \) from a given state

5 Experiment

• Excitation to Rydberg state from ground state in ultracold gases or BECs with sufficiently narrow and intense lasers

• Detection via field ionisation

• Realisation in Stuttgart with two-photon excitation (consider selection rules) without populating intermediate state (highly detuned)