Rydberg atoms

1 Definition and characteristics

Definition

- Rydberg atoms: At least 1 electron with high principle quantum number
- This electron is far away from the core compared to the ground state atoms
- Correspondence principle $\rightarrow$ Bohr's model: Estimation of properties

Size	$r = a_0 \cdot n^2 \sim n^2$
Energy	$E = \frac{-E_R}{n^2} \sim n^{-2}$
Level spacing	$\Delta E \sim n^{-3}$
Geom. cross section	$\sigma = \pi r^2 \sim n^4$
Dipole moment	$d = e \cdot r \sim n^2$
Polarizability	$\alpha = \frac{d^2}{\Delta E} \sim n^7$
vdW parameter	$C_6 = \frac{-(d_1 \cdot d_2)^2}{\Delta E} \sim n^{11}$
Radiative lifetime	$\sim n^3$

Limits of model:

- Low spatial overlap between core and outer electron required for agreement with model
- Good approximation for high principle quantum number, high angular momenta
- Description by empirical formula:

$$E_B = -\frac{E_{R,2}}{(n - \delta_{n,l})^2} \tag{1}$$

2 Interactions

- High sensitivity to electric fields (large dipole moments): Stark effect, ionisation
- Atom light interaction using a two-level atom with a semiclassical model leads to Hamiltonian:

$$H = -\frac{\hbar\Delta}{2}(1 - \sigma_z) + \hbar\Omega\sigma_x \tag{2}$$

  with σ being the Pauli matrices
- Interaction between Rydberg atoms via dipole dipole interaction $\sim \frac{C_3}{r^3}$ and van der Waals interaction $\sim \frac{C_6}{r^6}$
- Collectivity effects lead to factor $\sqrt{N}$ in Rabi excitation

Treatment in pair state model leads to different regimes:

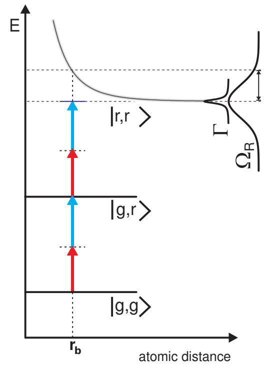
- Dipole dipole interactions for low energy level differences, even Förster resonances for vanishing energy difference
- Crossover between dipole dipole and van der Waals interactions for $U(r) \approx \Delta$
- Van der Waals interactions for high energy differences $\rightarrow$ Rydberg blockade

Rydberg blockade:

- Compare spatial dependant energy shift with line broadening effects
- Inside Rydberg blockade sphere, second Rydberg excitation is shifted out of resonance
- Blockade radius: Biggest radius in which only one excitation is possible (at fixed laser frequencies)

- Evaluation: Comparing interaction strength and laser excitation given by Rabi frequency

$$Z\frac{C_6}{r_B^6} = ZC_6 n_r^2 = \sqrt{N}\hbar\Omega_R \tag{3}$$



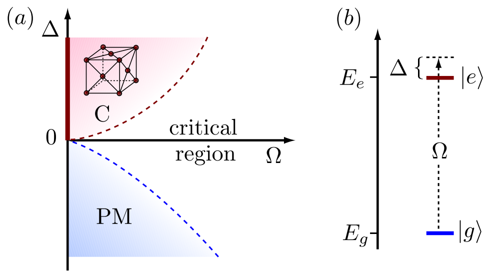
3 Universality

- Concept of universality can also be applied to Rydberg atoms
- Order parameter $f = \left\langle P_{rr}^{(i)} \right\rangle = \frac{N_R}{N_g}$, diverging critical length scale $\xi = \frac{a_R}{a_g} = \frac{1}{a_g}\sqrt{\frac{C_6}{\hbar\Omega_N}}$
- Parameters: $\alpha = \frac{\hbar\Omega_0}{C_6 n_g^2} = \frac{couplingstrength}{interaction}$, $\Delta' = \frac{\hbar\Delta}{C_6 n_g^2}$
- Phase transition from no Rydberg excitation (paramagnetic) to crystal given by blockade spheres (ferromagnetic)

Total Hamiltonian:

$$\hat{H} = -\frac{\hbar\Delta}{2}\sum_i (1-\sigma_z^{(i)}) + \frac{\hbar\Omega}{2}\sum_i \sigma_x^{(i)} + C_6 \sum_{j<i} \frac{P_{rr}^{(i)} P_{rr}^{(j)}}{|r_i - r_j|^6} \tag{4}$$

with $P_{rr}^{(i)} = |r_i\rangle\langle r_i| = \frac{1-\sigma_z^{(i)}}{2}$



4 Phase gate

- Rydberg atoms useful for quantum computation because of collectivity behaviour and control with Rydberg blockade
- Usage of π-pulses in combination with Rydberg blockade effect (different states involved) offers realisation of phase gate, e.g. a phase shift of π from a given state

5 Experiment

- Excitation to Rydberg state from ground state in ultracold gases or BECs with sufficiently narrow and intense lasers
- Detection via field ionisation
- Realisation in Stuttgart with two-photon excitation (consider selection rules) without populating intermediate state (highly detuned)