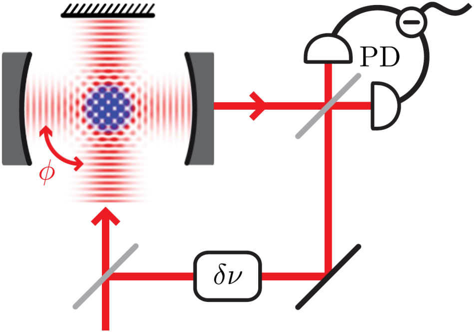
Cavity Quantum-Electrodynamics (Cavity QED)

Tobias Bäuerle

Hauptseminar physics of cold gases

9th of July, 2013

Experimental cavity QED setup



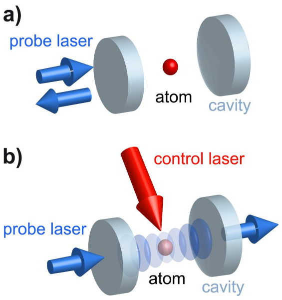
Dicke phase transition of a BEC in a cavity [1]

[1] PRL 107, 140402 (2011)

Outline

Motivation

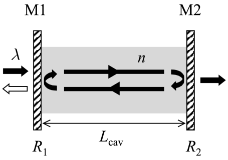
- Changed properties of atom-light interactions in a cavity
- Modified spontaneous emission rate
  → performance of a laser medium
  → laser cooling efficiency
- Single-photon phase gates for quantum computation
- Dicke phase transition
  → research on supersolid phases



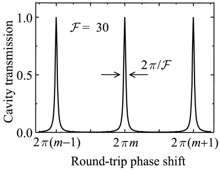
EIT inside a cavity[2]

[2] Website Max-Planck-Institute for quantumoptics, researchdetails

Light in a cavity



Resonator with two planar mirrors [3]



Transmission through a resonator [4]

- Transmission $T = \frac{1}{1+4\frac{\mathcal{F}^2}{\pi^2}\sin^2\left(\frac{\Phi}{2}\right)}$

  with Phase $\Phi = \frac{4\pi n L_{cav}}{\lambda}$

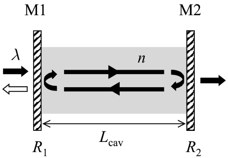
  and Finesse $\mathcal{F} = \frac{\pi (R_1 R_2)^{1/4}}{1-\sqrt{R_1 R_2}} = \frac{\omega_{FSR}}{\Delta\omega}$

- Resonance condition: $L_{cav} = \frac{\lambda m}{2n}$, $m \in \mathbb{N}$

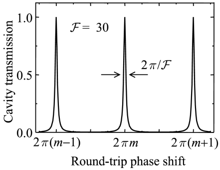
- Quality factor: $Q = \frac{\omega}{\Delta\omega}$

[3,4] Mark Fox: Quantum Optics, Oxford university press (2006)

Light in a cavity



Resonator with two planar mirrors [3]



Transmission through a resonator [4]

- Chance for photon loss on a mirror: $(1-R)$
- Travel time between 2 reflections: $t = L_{cav}\frac{n}{c}$
- Photon lifetime: $\tau = \frac{L_{cav}\cdot n}{(1-R)\cdot c}$
- Decay rate: $\kappa = \frac{1}{\tau}$ $(= \Delta\omega)$

[3,4] Mark Fox: Quantum Optics, Oxford university press (2006)

Atom-light interaction

Coupling parameter g_0

- Dipole interactions between atom and light field:

  $$\Delta E = \langle 1 | (\vec{d} \cdot \vec{E}) | 2 \rangle$$

- Dipole matrix element:

  $$\mu_{12} = e \langle 1 | \vec{r} | 2 \rangle$$

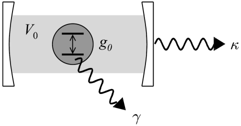
- Coupling parameter g_0 from interaction with vacuum field:

  $$g_0 = \frac{1}{\hbar} \Delta E_{vac} = \frac{1}{\hbar} \langle 1 | (\vec{d} \cdot \vec{E}_{vac}) | 2 \rangle = \frac{1}{\hbar} |\mu_{12} \cdot \mathcal{E}_{vac}| = \frac{1}{\hbar} \left| \mu_{12} \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} \right|$$

Atom-light interaction

Different regimes

- Three parameters control the behaviour
  - Cavity loss $\kappa = \frac{\omega}{Q}$
  - Non-resonant decay γ
  - Coupling parameter g_0
- Weak coupling regime:
  $g_0 \ll \max(\kappa, \gamma)$, photon emission irreversible
- Strong coupling regime:
  $g_0 \gg \max(\kappa, \gamma)$, photon emission reversible
- Normally strong coupling for $Q \gg \sqrt{\frac{2\varepsilon_0 \hbar\omega V_0}{\mu_{12}}}$



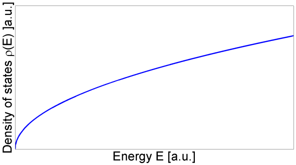
Parameters of a cavity system [5]

[5] Mark Fox: Quantum Optics, Oxford university press (2006)

Atom-light interaction

Weak coupling regime

- Density of states:
$$\rho(E) = \sum_k \delta(E - E(\vec{k})) = \frac{V}{(2\pi)^3} \int \mathrm{d}^3\vec{k}\,\delta(E - E(\vec{k}))$$
- Free electrons:
$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow \rho(E) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \sqrt{E}$$



Density of states, free electrons

Atom-light interaction

Weak coupling regime

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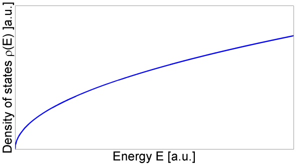
- Photons in free space:

$$E = \hbar c k \Rightarrow \rho_{\text{free}}(E) = \frac{V}{\pi^2 \hbar^3 c^3} E^2$$

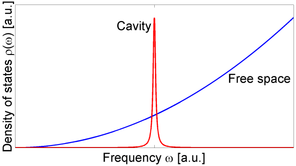
$$\rho_{\text{free}}(\omega) = \frac{V}{\pi^2 c^3} \omega^2$$

- Photons in a cavity:

$$\rho_{\text{cav}}(\omega) = \frac{2}{\pi \Delta\omega_c} \frac{\Delta\omega_c^2}{4(\omega - \omega_c)^2 + \Delta\omega_c^2}$$



Density of states, free electrons



Density of states, photons

Atom-light interaction

Weak coupling regime

- Fermi's golden rule:
  $W_{i\to f} = \frac{2\pi}{\hbar^2} |\langle f| H_{\text{int}} |i\rangle|^2 \delta(\omega - \omega_{fi})$
- Spontanous emission of the excited state:
  $W_{i\to f} = \frac{2\pi}{\hbar^2} |M_{12}|^2 \rho(\omega)$ with
  $|M_{12}|^2 = \left\langle \vec{p} \cdot \vec{\mathcal{E}} \right\rangle = \xi^2 \mu_{12}^2 \mathcal{E}_{\text{vac}}^2 = \xi^2 \frac{\mu_{12}^2 \hbar\omega}{2\varepsilon_0 V_0}$
- In free space:
  $W_{\text{free}} = \frac{\mu_{12}^2}{3\pi\varepsilon_0 \hbar c^3} \omega^3$

  In a cavity:
  $W_{\text{cav}} = \frac{2Q\mu_{12}^2}{\varepsilon_0 \hbar V_0} \xi^2 \frac{\Delta\omega_c^2}{4(\omega_0 - \omega_c)^2 + \Delta\omega_c^2}$

Atom-light interaction

Weak coupling regime

- $W_{\text{free}} = \frac{\mu_{12}^2}{3\pi\varepsilon_0\hbar c^3}\omega^3$

  $W_{\text{cav}} = \frac{2Q\mu_{12}^2}{\varepsilon_0\hbar V_0}\xi^2\frac{\Delta\omega_c^2}{4(\omega_0-\omega_c)^2+\Delta\omega_c^2}$

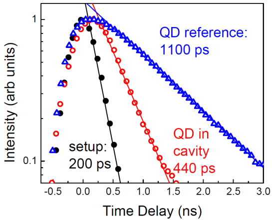
- Purcell factor:

  $F_p = \frac{W_{\text{cav}}}{W_{\text{free}}} = \frac{3Q(\lambda/n)^3}{4\pi^2 V_0}\xi^2\frac{\Delta\omega_c^2}{4(\omega_0-\omega_c)^2+\Delta\omega_c^2}$

  on resonance $\Rightarrow$ $F_p = \frac{3Q(\lambda/n)^3}{4\pi^2 V_0}$

- $F_p > 1$: Spontaneous emission enhanced, reduced lifetime

- $F_p < 1$: Spontaneous emission suppressed, longer lifetime



Purcell effect of a single quantum dot[6]

[6] Proc. of SPIE Vol. 6101, 61010W, (2006)

Atom-light interaction

Strong coupling regime

- Jaynes-Cummings-Hamiltonian:

  $\hat{H}_{JC} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_3 + \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right)$

- Eigenenergies of the system:

  $E_{1,2} = \left(n + \frac{1}{2}\right)\hbar\omega \pm \frac{\hbar}{2}\sqrt{\Delta^2 + 4\lambda^2\left(n+1\right)}$

- Energie differences:

  $\Delta E = \hbar\sqrt{\Delta^2 + 4\lambda^2\left(n+1\right)}$

Atom-light interaction

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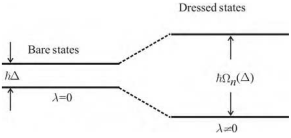


Energy splitting of the dressed states[7]

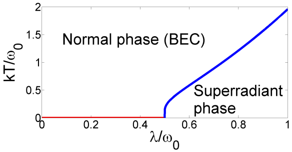
[7] C. C. Gerry and P. L. Knight: Introductory Quantum Optics, Cambridge university press (2005)

Dicke phase transition

Theoretical description

- Dicke Hamiltonian:
  $\hat{H}_D = \hbar\omega_0 \hat{J}_z + \hbar\omega \hat{a}^\dagger \hat{a} + \frac{2\hbar\lambda}{\sqrt{N}} \left(\hat{a}^\dagger + \hat{a}\right) \hat{J}_x,$
  with collective atomic dipole J
- Second order phase transition for
  $\lambda > \lambda_{cr} = \frac{\sqrt{\omega\omega_0}}{2},\ T = 0$
- Parity symmetry under
  $\left(\hat{a}, \hat{J}_x\right) \rightarrow \left(-\hat{a}, -\hat{J}_x\right)$ broken
- Superradiant phase:
  Coherent interaction of an atomic ensemble via a lightfield

- Normal phase:
  $\langle\hat{a}\rangle = 0,\ \langle\hat{J}_x\rangle = 0,$
  superradiant phase:
  $\langle\hat{a}\rangle \neq 0,\ \langle\hat{J}_x\rangle \neq 0$

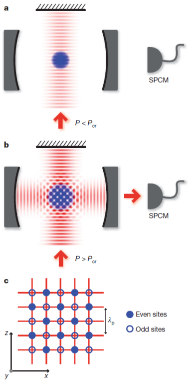


Critical temperature in dependence of the coupling parameter for the superradiant phase

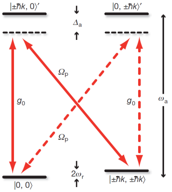
Dicke phase transition

Experimental realisation: Esslinger group (ETH Zürich) 2010

- ^{87}Rb-BEC in a ultrahigh-finesse cavity ($L_{\text{cav}} = 176\,\mu\text{m}$)
- Offresonant coupling laser: $\lambda_p = 784.5\,\text{nm}$
- Raman transition paths



BEC in a cavity for different pump powers[8]



Raman channels of the scattering process[9]

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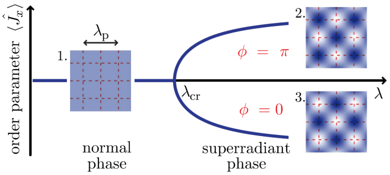
[8] Nature 464, 1301–1306 (29 April 2010)

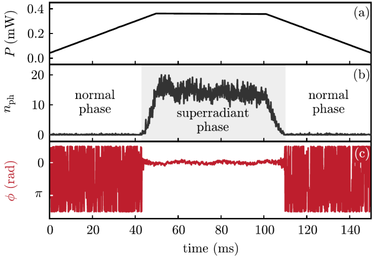
[9] PRL 107, 140402 (2011)

Dicke phase transition

Experimental realisation: Esslinger group (ETH Zürich) 2010



Normal and superradiant phase of the BEC[10]



Pump power, intracavity photons and their phase depending of the time[11]

- Cavity mode builds up $\rightarrow$ 2 sublattices
- Order in BEC density

- Both states measured

[10,11] PRL 107, 140402 (2011)

Summary

- Single-mode cavity changes properties of atoms in it
- Weak-coupling regime: Change of spontaneous emission rate of excited states
- Strong-coupling regime: Energy shift of the dressed states
- Dicke phase transition: BEC undergoes phase transition to a supersolid state

Thank you for your attention!