Hauptseminar physics of cold gases
Cavity Quantum-Electrodynamics (QED)

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LIGHT IN A CAVITY

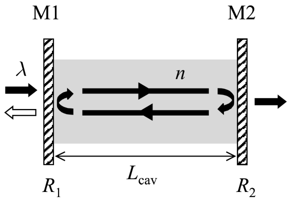

Figure 1. Resonator with two planar mirrors R_1 and R_2.[1]

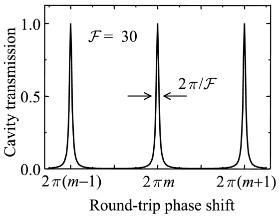

Figure 2. Transmission of different resonant modes through a resonator. [1]

- Transmission:
  $T = \frac{1}{1+4\frac{\mathcal{F}^2}{\pi^2}\sin^2\left(\frac{\Phi}{2}\right)}$ with phase Φ and Finesse $\mathcal{F}$
- Resonance condition:
  $L_{\text{cav}} = \frac{\lambda m}{2n}$, $m \in \mathbb{N}$, n: refractive index
- Quality factor:
  $Q = \frac{\omega}{\Delta\omega}$
- Photon decay rate:
  $\kappa = \frac{1}{\tau} = \frac{(1-R)\cdot c}{L_{\text{cav}}\cdot n} \, (= \Delta\omega)$

ATOM-LIGHT INTERACTION

The coupling parameter g_0 is calculated from interaction between atom and vacuum field:

$$\begin{aligned} g_0 &= \frac{1}{\hbar}\Delta E_{\text{vac}} = \frac{1}{\hbar}\left\langle 1\right| (\vec{d}\cdot\vec{E}_{\text{vac}}) \left|2\right\rangle = \frac{1}{\hbar}|\mu_{12}\cdot\mathcal{E}_{\text{vac}}| \\ &= \frac{1}{\hbar}\left|\mu_{12}\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\right| \end{aligned}$$

with the dipole matrix element $\mu_{12} = e\left\langle 1\right| \vec{r} \left|2\right\rangle$.

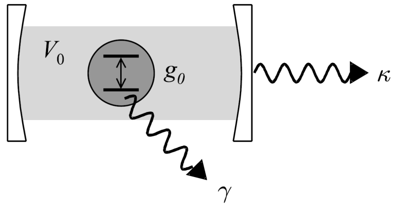

Figure 3. Parameters of a cavity system: mode volume V_0, cavity loss κ, non-resonant decay γ and coupling parameter g_0. [1]

- Weak coupling regime:
  $g_0 \ll \max(\kappa, \gamma)$, irreversible photon emission
- strong coupling regime:
  $g_0 \gg \max(\kappa, \gamma)$, reversible photon emission

Weak coupling regime

- Density of states:
  $\rho(E) \widehat{=} \sum_k \delta(E - E(\vec{k})) = \frac{V}{(2\pi)^3}\int \mathrm{d}^3\vec{k}\,\delta(E - E(\vec{k}))$
- Spontaneous emission given by Fermi's golden rule:
  $W_{i\to f} = \frac{2\pi}{\hbar^2}|M_{12}|^2\rho(\omega)$ with
  $|M_{12}|^2 = \left\langle \vec{p}\cdot\vec{\mathcal{E}}\right\rangle = \xi^2\mu_{12}^2\mathcal{E}_{\text{vac}}^2 = \xi^2\frac{\mu_{12}^2\hbar\omega}{2\varepsilon_0 V_0}$
- In free space:
  $\rho_{\text{free}} = \frac{V}{\pi^2 c^3}\omega^2$
  $W_{\text{free}} = \frac{\mu_{12}^2}{3\pi\varepsilon_0\hbar c^3}\omega^3$
- In a cavity:
  $\rho_{\text{cav}}(\omega) = \frac{2}{\pi\Delta\omega_c}\frac{\Delta\omega_c^2}{4(\omega-\omega_c)^2+\Delta\omega_c^2}$
  $W_{\text{cav}} = \frac{2Q\mu_{12}^2}{\varepsilon_0\hbar V_0}\xi^2\frac{\Delta\omega_c^2}{4(\omega_0-\omega_c)^2+\Delta\omega_c^2}$

In the weak coupling regime the spontaneous emission rate is changed by the Purcell factor $F_p = \frac{W_{\text{cav}}}{W_{\text{free}}}$.

Strong coupling regime

- Description with the Jaynes-Cummings-Hamiltonian:

$$\begin{aligned} H_{\text{JC}} &= H_{\text{photon}} + H_{\text{atom}} + H_{\text{int}} \\ &= \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_3 + \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right) \end{aligned}$$

$\hat{a}^\dagger$, $\hat{a}$: Creation and annihilation operator of the light field
$\hat{\sigma}_+$, $\hat{\sigma}_-$: Atomic transition operator
$\hat{\sigma}_3$: Atomic inversion operator

- Dressed states:
  $|e, n\rangle$ and $|g, n+1\rangle$
- Eigenenergies:
  $E_{1,2} = \left(n + \frac{1}{2}\right)\hbar\omega \pm \frac{\hbar}{2}\sqrt{\Delta^2 + 4\lambda^2\,(n+1)}$
  $\Delta = \omega_0 - \omega$: Detuning of the system
- Energy splitting:
  $\Delta E = \hbar\sqrt{\Delta^2 + 4\lambda^2\,(n+1)}$

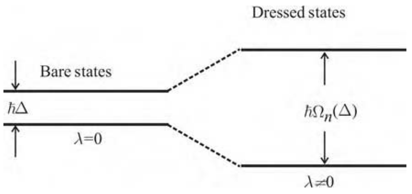

Figure 4. Energy splitting in a coupled atom-light system. [2]

DICKE PHASE TRANSITION

Theoretical description

- Superradiant phase:
  Coherent interaction of an atomic ensemble via a lightfield
- Description with the Dicke-Hamiltonian:
  $H_{\mathrm{D}} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\hat{J}_z + \frac{2\hbar\lambda}{\sqrt{N}}\left(\hat{a}^\dagger + \hat{a}\right)\hat{J}_x$
  J: collective dipole operator
- Second order phase transition for sufficient coupling
  $\lambda > \lambda_{\mathrm{cr}} = \frac{\sqrt{\omega\omega_0}}{2}$, $T = 0$
- Possible order parameters:
  $\langle\hat{a}\rangle = 0$, $\langle\hat{J}_x\rangle = 0$ in the normal phase
  $\langle\hat{a}\rangle \neq 0$, $\langle\hat{J}_x\rangle \neq 0$ in the superradiant phase

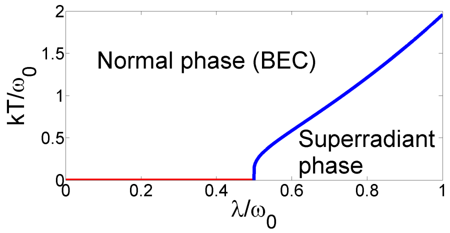

Figure 5. Phase diagram depending on the coupling strength and the temperature.

Experimental realisation

Experiment carried out by the group of Tilman Esslinger (ETH Zürich) in 2010

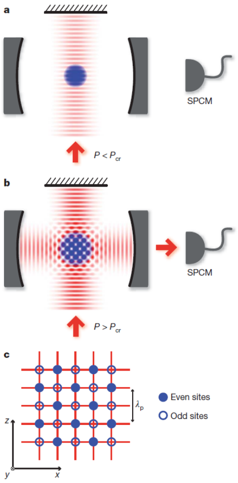

Figure 6. BEC inside an ultrahigh-finesse cavity for different pump powers (a,b). c shows the structure for both possible superradiant phases.[3]

- ^{87}Rb-BEC in an ultrahigh-finesse cavity with length $L_{\mathrm{cav}} = 176\,\mu$m.
- Far-detuned (4.5 nm) pump laser drives Raman transitions in the BEC.
- For sufficient pump power a cavity field builds up → order in BEC density.

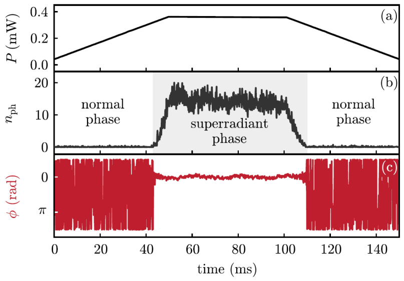

Figure 7. Measured phase transition of the BEC.[4]

[1] Mark Fox: *Quantum Optics*, Oxford University Press (2006)
[2] C.C.Gerry, P.L.Knight: *Introductory Quantum Optics*, Cambridge University Press (2005)
[3] Nature 464, 1301-1306 (29. April 2010)
[4] PRL 107, 140402 (2011)