Alkaline-earth SU(N)

Physics of cold gases

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16.07.2013

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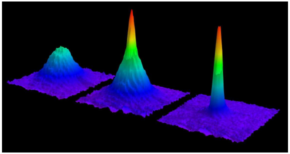
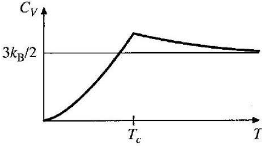
Motivation

Bose-Einstein condensation

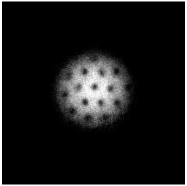


Velocity distribution of atoms for different temperatures[1]

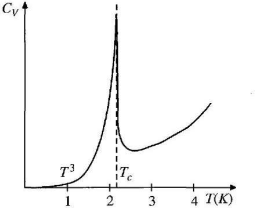


Heat capicity of an ideal Bose-Einstein gas[2]

Superfluidity



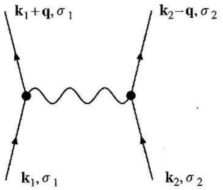
Vortices in a rotating BEC[1]



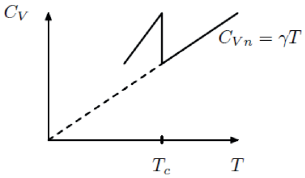
^{4}He heat capicity[2]

Motivation

- **Superconductivity**



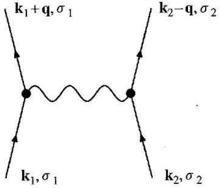
Interaction of electrons via exchange of a phonon[2]



Heat capicity of a superconducter resulting from the Ginzburg-Landau theory[2]

Motivation

- **Superconductivity**



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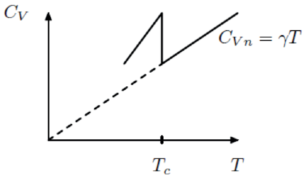

Heat capicity of a superconducter resulting from the Ginzburg-Landau theory[2]

Is it possible to simulate solid state physics with cold gases?

Motivation

Properties of alkaline-earth metals:

- Two valence electrons
- Fermionic and bosonic isotopes

Period ↓ / Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Periodic table of elements[3]

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Periodic table of elements[3]

$\Rightarrow$ Two key features:

- Ultranarrow doubly forbidden transition between the ground state $^1S_0$ and the state $^3P_0$
- Fermionic isotopes show almost perfect decoupling of the electronic angular moment J and the nuclear spin I for these two states

Outline - Alkaline-earth SU(N)

Time definition and clock quantities

- *The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*[4]

- The oscillator is characterized by the Q-factor:

$$Q = \frac{\nu_0}{\delta\nu}$$

- The fractional instability is given by the Allan deviation:

$$\sigma \approx \frac{1}{Q\sqrt{N_{\text{at}} \cdot \tau}}$$

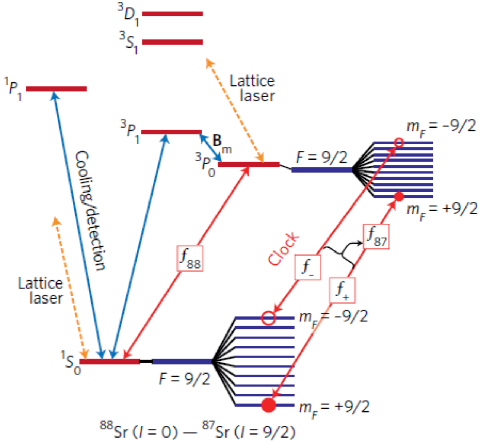
Time definition and clock quantities

- Possible improvements:
  - Switching from a transition with radio-frequency ν_{RF} $(10^9\,\mathrm{Hz})$ to one with higher frequency, e.g. optical frequency ν_{OF} $(10^{14}\,\mathrm{Hz})$
  - Using a transition with a small linewidth $\delta\nu$
  - Cooling atoms to eliminate Doppler-broadening $(\delta\nu_{\mathrm{Doppler}} = -\frac{v}{c}\nu_0)$ and to increase the investigation time
  - Trapping atoms in an optical lattice to interrogate them simultaneously

$\Rightarrow$ Cs atomic clock: $\sigma = 10^{-15}$

$\Rightarrow$ Optical lattice clock: $\sigma = 10^{-18}$

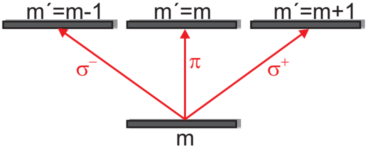
Strontium level scheme



Level scheme for a strontium optical lattice clock[5]

Selection rules

- Dipole allowed transitions:
  - $\Delta n =$ arbitrary
  - $\Delta l = \pm 1$
  - $\Delta J = 0, \pm 1$
  - $J = 0 \nrightarrow J' = 0$



with linearly ($\Delta m_J = 0$) and circularly ($\Delta m_J = \pm 1$) polarized light

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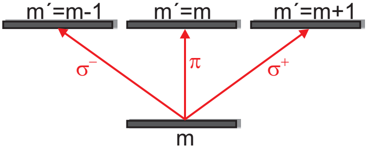

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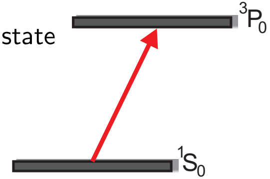
- For atoms with strong LS-coupling:
  - $\Delta S = \pm 1$
  - $\Delta L = 0, \pm 1, \pm 2$

$\Rightarrow$ Transitions between singlet and triplet states are allowed

The $^1S_0$ - $^3P_0$ transition

- The clock transition $^1S_0 \rightarrow {}^3P_0$ is doubly forbidden
  - Transition from a spin-singlet to a spin-triplet state
  - $J = 0 \rightarrow J' = 0$



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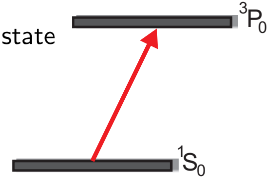
  - Hyperfine interactions admixe the $^3P_1$-state
  - Typical linewidth $\approx 10\,\mathrm{mHz}$



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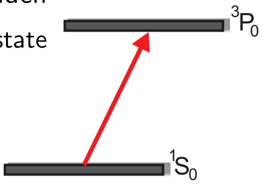
- Fermionic isotope (^{87}Sr,$I = \frac{9}{2}$):
  - Hyperfine interactions admixe the $^3P_1$-state
  - Typical linewidth $\approx 10\,\mathrm{mHz}$
- Bosonic isotope (^{88}Sr,$I = 0$):
  - No hyperfine interactions $\rightarrow$ transition is strongly suppressed
  - $^3P_1$-state can be admixed by an external magnetic field



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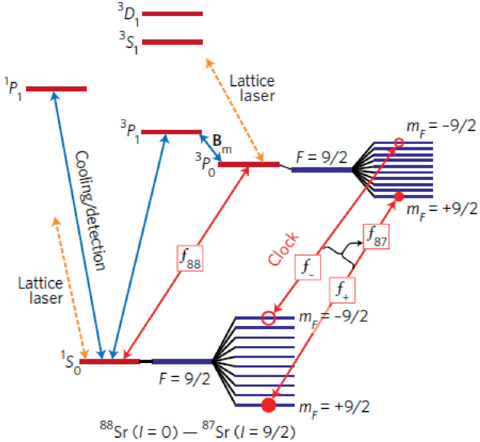
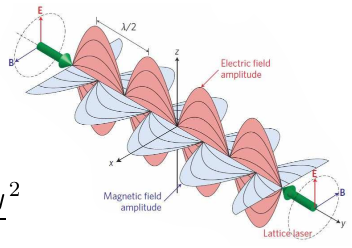


Level scheme for a strontium optical lattice clock[5]

Optical lattices

- Superposition of two laser beams with wavelength λ_L
- Atoms experience an optical potential:

$$V(y,r) = -\alpha(\omega_\mathrm{L}) E_\mathrm{L}^2 e^{-2\left(\frac{r}{w(y)}\right)^2} \cos \frac{2\pi y}{\lambda_\mathrm{L}}^2$$



1D optical lattice scheme [5]

- Dipole force

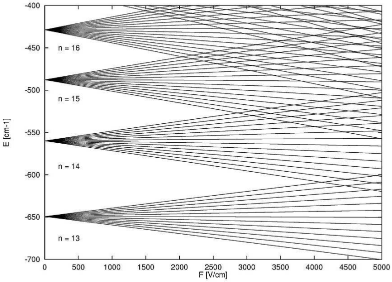
$$F_\mathrm{dip} = -\nabla V(y,r)$$

points to

  - the intensity maxima for red detuned light ($\lambda_\mathrm{L} \geq \lambda_\mathrm{R}$)
  - the intensity minima for blue detuned light ($\lambda_\mathrm{L} \leq \lambda_\mathrm{R}$)

Stark effect

- Atomic energy levels get split and shifted due to an external electric field
- The Zeeman effect is the magnetic analogue
- The resulting clock frequency is:



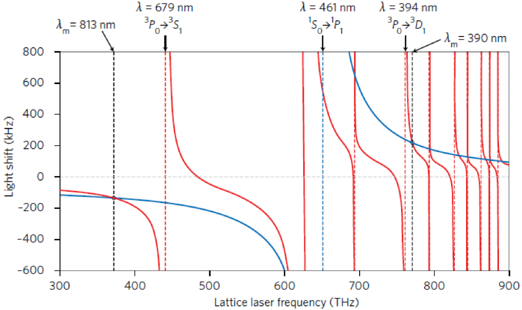
Stark map for $13 \leq n \leq 16$ of hydrogen[6]

$$\nu = \nu_0 - [\alpha_{\mathsf{e}}(\omega_{\mathsf{L}}) - \alpha_{\mathsf{g}}(\omega_{\mathsf{L}})] \frac{E_{\mathsf{L}}^2}{4h} + O(E_{\mathsf{L}}^4)$$

with the polarizability

$$\alpha_a(\omega_{\mathsf{L}}) = \sum_b \frac{|\langle a|\underline{D} \cdot \underline{\epsilon}|b\rangle|^2}{E_b - E_a - \omega_{\mathsf{L}}} + \sum_b \frac{|\langle a|\underline{D} \cdot \underline{\epsilon}|b\rangle|^2}{E_b - E_a + \omega_{\mathsf{L}}}$$

Magic Wavelengths

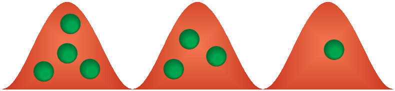


Light shifts for the ${}^1S_0$- (blue line) and ${}^3P_0$- (red line) states depending on the lattice laser frequency[5]

- The polarizabilities are the same for both clock states $(\alpha_{\mathsf{e}}(\omega_{\mathsf{L}}) = \alpha_{\mathsf{g}}(\omega_{\mathsf{L}}))$
- The ac-Stark effect is canceled out
- Both states experience the same trapping potential

Blue and red magic lattices

- Red magic lattice:
  - Atoms get trapped in areas of high laser intensities
  - Higher order light shifts can not be neglected
  - Reduction of laser intensity $\rightarrow$ spectral line broadens and collision rate increases



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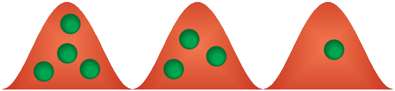
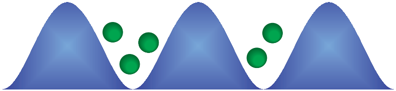


- Blue magic lattice:
  - Atoms get trapped in areas with low laser intensities
  - Contributions of higher order light shifts can be neglected



Outline - Alkaline-earth SU(N)

Bloch functions

- The optical lattice potential is periodic

$$V_{\mathsf{L}}(\underline{r}) = V_{\mathsf{L}}(\underline{r} + \underline{a}_j)$$

with the lattice vector $\underline{a} = \{a_\alpha\}$ and $\underline{a}_j = \{n_{\mathrm{j}} a_\alpha | n_{\mathrm{j}} = 0, \pm 1, ...\}$

- Solutions for periodic potentials are Bloch functions

$$\phi_{nk}(\underline{r}) = f_{nk}(\underline{r}) e^{i\underline{k}\cdot\underline{r}}$$

where $f_{nk}(\underline{r}) = f_{nk}(\underline{r} + \underline{a}_j)$

- The quasimomentum $\underline{k}$ corresponds to the Brillouin zone

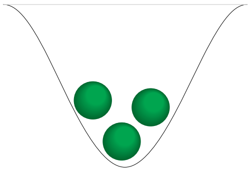
$$B = \{\underline{k} : -\frac{\pi}{a_\alpha} \leq k_\alpha \leq \frac{\pi}{a_\alpha}\}$$

Wannier functions

- For strong localization the Wannier functions

$$\omega_{nj}(\underline{r}) = \frac{1}{\sqrt{N_\mathrm{L}}} \sum_k \phi_{nk}(\underline{r}) e^{-i\underline{k}\cdot\underline{a}_j}$$

can be used as a basis



- Orthonormality:

$$\int \omega^*_{mi}(\underline{r})\omega_{nj}(\underline{r})\mathrm{d}\underline{r} = \delta_{mn}\delta_{ij}$$

- Real:

$$\omega_{nj}(\underline{r}) = \omega^*_{nj}(\underline{r})$$

- Completeness:

$$\sum_{nj} \omega_{nj}(\underline{r})\omega^*_{nj}(\underline{r}') = \delta(\underline{r}-\underline{r}')$$

- Periodicity:

$$\omega_n(\underline{r}) = \omega_n(\underline{r} - \underline{a}_j)$$

Hamiltonian for trapped fermions in a 2D-lattice

- Fermionic alkaline-earth atoms trapped in the lowest band of an optical 2D-lattice are described by:

$$
\begin{aligned}
H = & \sum_{\alpha m} \int \mathrm{d}^3 \underline{r}\, \psi^\dagger_{\alpha m}(\underline{r}) [-\frac{\hbar^2}{2M} \nabla^2 + V_\alpha(\underline{r})] \psi_{\alpha m}(\underline{r}) \\
& + \hbar\omega_0 \int \mathrm{d}^3 \underline{r} [\rho_e(\underline{r}) - \rho_g(\underline{r})] + \frac{g^+_{eg} + g^-_{eg}}{2} \int \mathrm{d}^3 \underline{r}\, \rho_e(\underline{r}) \rho_g(\underline{r}) \\
& + \sum_{\alpha, m<m'} g_{\alpha\alpha} \int \mathrm{d}^3 \underline{r}\, \rho_{\alpha m}(\underline{r}) \rho_{\alpha m'}(\underline{r}) \\
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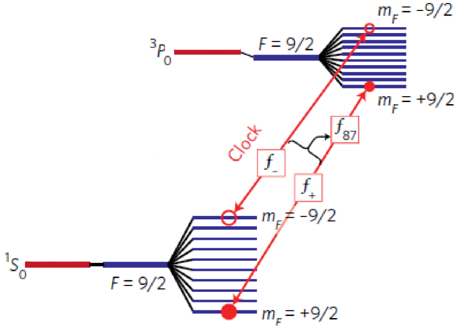

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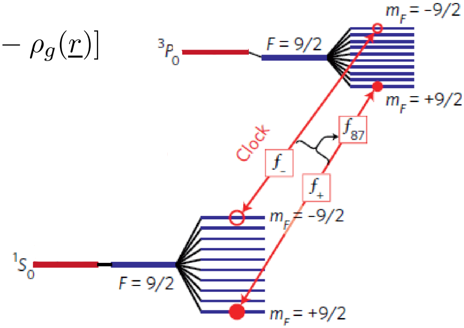

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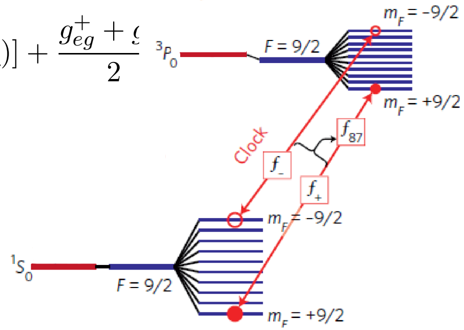

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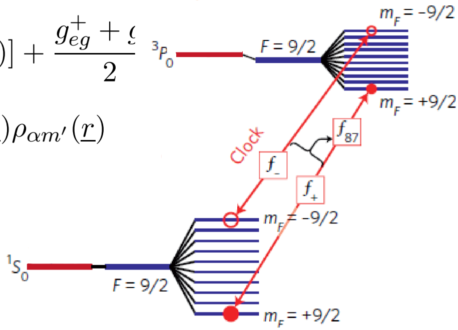

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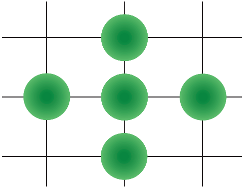
Two-orbital single-band Hubbard Hamiltonian

- The resulting Hamiltonian is

$$H = - \sum_{\langle j,i \rangle \alpha, m} J_\alpha (c^\dagger_{i\alpha m} c_{j\alpha m} + h.c.)$$

with the energies

- J_α: Tunnelling between two neighbouring lattice sites



Two-orbital single-band Hubbard Hamiltonian

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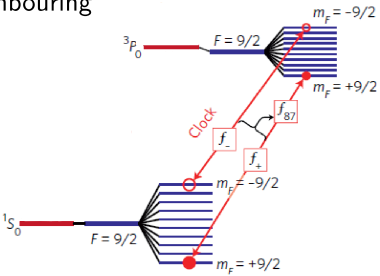
$$H = -\sum_{\langle j,i\rangle \alpha,m} J_\alpha (c^\dagger_{i\alpha m} c_{j\alpha m} + h.c.) + \sum_{j,\alpha} \frac{U_{\alpha\alpha}}{2} n_{j\alpha}(n_{j\alpha} - 1)$$

with the energies
- J_α: Tunnelling between two neighbouring lattice sites
- $U_{\alpha\alpha}$: Onsite interactions



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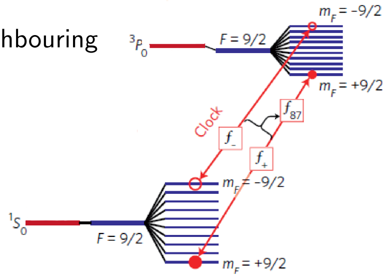
$$+ V \sum_j n_{je} n_{jg}$$

with the energies

- J_α: Tunnelling between two neighbouring lattice sites
- $U_{\alpha\alpha}$: Onsite interactions
- V: Direct interactions



Two-orbital single-band Hubbard Hamiltonian

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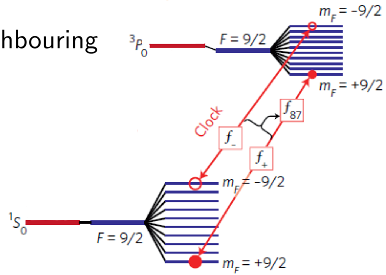
$$+ V \sum_j n_{je} n_{jg} + V_{ex} \sum_{j,m,m'} c^\dagger_{jgm} c^\dagger_{jem'} c_{jgm'} c_{jem}$$

with the energies

- J_α: Tunnelling between two neighbouring lattice sites
- $U_{\alpha\alpha}$: Onsite interactions
- V: Direct interactions
- V_{ex}: Exchange interactions



Outline - Alkaline-earth SU(N)

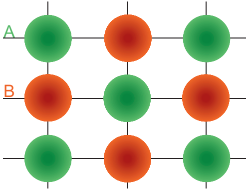
Limits of the two-orbital Hamiltonian

Two-orbital Hamiltonian

U(1)xSU(N)-symmetry

Kugel-Khomskii model

- Strongly interacting regime ($\frac{J}{U} \ll 1$)
  $\rightarrow$ Effective spin Hamiltonian
- Special case:
  only $|g\rangle$-state atoms in a bipartite lattice



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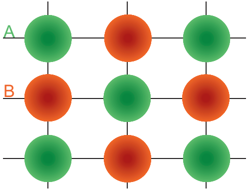


- Kugel-Khomskii Hamiltonian

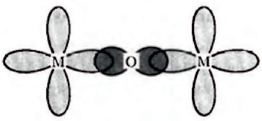
$$H = \frac{2J_g^2 U_{gg}}{U_{gg}^2 - (U_{gg}(n_{\mathsf{A}} - n_{\mathsf{B}}) + \Delta)^2} \sum_{\langle i,j \rangle} S_{ij}^2$$

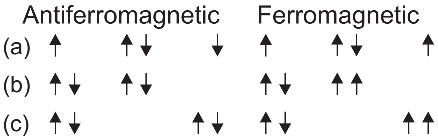
where

$$S_{ij}^2 = \sum_{mn} S_m^n(i) S_n^m(j) = \sum_{mn} c_{im}^\dagger c_{in} c_{jn}^\dagger c_{jm}$$

Kugel-Khomskii model

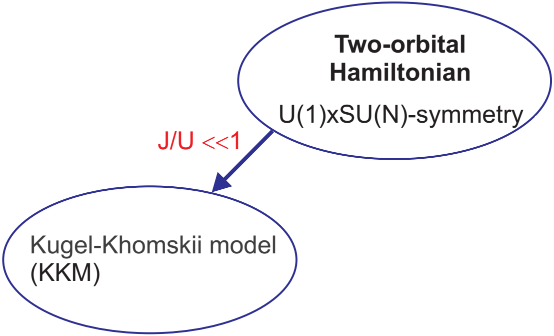
- KKM describes the magnetic properties of transition-metal compounds based on superexchange
- Superexchange is the strong coupling between two magnetic non-neighbouring ions through a non-magnetic ion
  - Ferromagnetic interactions, e.g. for a $90\,^\circ$-angle between both magnetic ions
  - Otherwise antiferromagnetic interactions





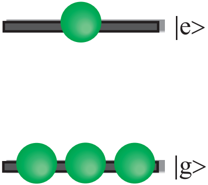
Transition metal oxide with possible spin configurations[7]

Limits of the two-orbital Hamiltonian



Kondo lattice model

- Onsite interaction between ground state atoms is turned off
  $\rightarrow U_{gg} = 0$
- One atom in $|e\rangle$-state per lattice site
  $\rightarrow n_{je} = 1$
- Deep lattice such that $J_e \ll U_{ee}$
  $\rightarrow J_e = 0$



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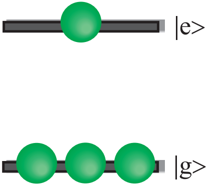


- Kondo lattice Hamiltonian

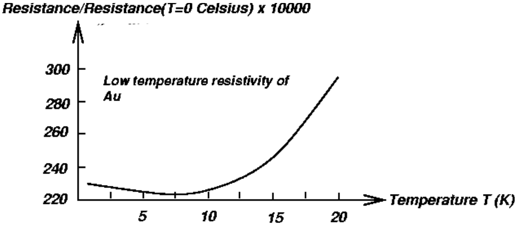
$$H = -\sum_{\langle i,j\rangle m} J_g(c^\dagger_{igm} c_{jgm} + h.c.) + V_{ex} \sum_{j,m,m'} c^\dagger_{jgm} c^\dagger_{jem'} c_{jgm'} c_{jem}$$

Kondo lattice model

- Kondo lattice model:
  - Conduction electrons are free electrons in the conduction band
  - Magnetic impurties at site j are localized spins
  - Spin-spin interactions between the conduction electrons and the impurites

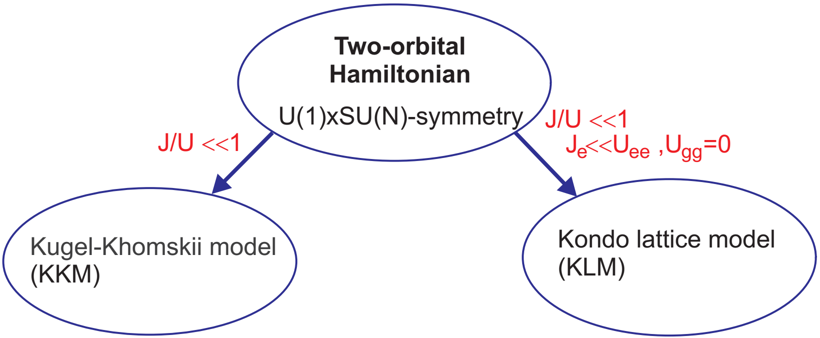
$\Rightarrow$ Explains the Kondo effect:

- Resistance minimum at a non-zero temperature
- For $V_{ex} < 0$: spin-antisymmetric states between conduction electrons and localized spins



Temperature dependent resistance of gold with impurities[8]

Limits of the two-orbital Hamiltonian



Conclusion

- The clock transition ${}^1S_0 \to {}^3P_0$ gets enabled due to
  - strong LS-coupling
  - the admixtur of the ${}^3P_1$-state
- Perturbations of the clock frequency due to the ac-Stark shift can be eliminate by
  - creating an optical lattice with one of the possible magic wavelengths
  - using a blue detuned magic wavelength
- Two-orbital single-band Hamiltonian

  $\Rightarrow$ Kugel-Khomskii Hamiltonian

  $\Rightarrow$ Kondo lattice Hamiltonian

Thank you for your attention!

Picture sources

[1] Ketterle group (http://cua.mit.edu/ketterle_group/Nice_pics.htm)

[2] James F. Annett, *Superconductivity, Superfluids and Condensates*, Oxforxd University Press 2012

[3] Wikipedia, (http://en.wikipedia.org/wiki/Periodic_table)

[4] http://www.bipm.org/en/si/si_brochure/chapter2/2-1/second.html

[5] Hidetoshi Katori, Nature Photonics 5, 203210 (31. March 2011)

[6] Wikipedia (https://en.wikipedia.org/wiki/File:Hfspec1.jpg)

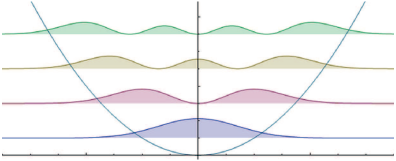
[7] Stephen Blundell, *Magnetism in condensed matter*, Oxforxd University Press 2001

[8] Wikipedia (http://en.wikipedia.org/wiki/File:Classickondo.png)

Lamb-Dicke regime

- The potential is harmonic near the bottom of each lattice site with the level spacing

$$E_{\mathrm{ho}} = \hbar \frac{2\pi}{\lambda_{\mathrm{L}}} \sqrt{\frac{2\alpha(\omega_{\mathrm{L}}) E_{\mathrm{L}}^2}{M}}$$



Probability of presence in a quantum HO

- By probig the clock transition the atoms get a momentum

$$p = \frac{\hbar \omega_{\mathrm{p}}}{c}$$

- Atoms stay in the same state of motion as long as $\frac{p^2}{2M} \leq E_{\mathrm{ho}}$

$\Rightarrow$ Lamb-Dicke regime ensures that the atomic motion does not modify the clock frequency

Atom-atom interactions

- Collisional frequency shift is related to the mean field energy shift

$$\delta E = \frac{4\pi\hbar^2 a\eta}{m} g^{(2)}(0)$$

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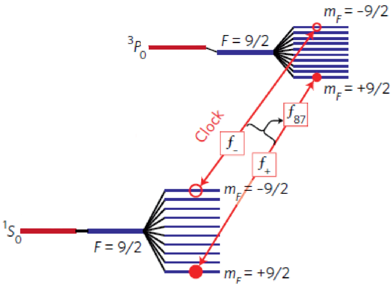
  $\rightarrow$ More complex lattice geometries, e.g. 3D-lattice, to minimize the frequency shift

- Indistinguishable fermions: $g^{(2)}(0) = 0$ due to the Pauli exclusion principle

  $\rightarrow$ Creation of spin-polarized fermions to achive $\delta E = 0$

Spin-polarization

- Atoms are prepared in the two outer Zeeman levels of the ground state by optical pumping
- $f_{\pm}$ of the corresponding transitions $^1S_0(F = \frac{9}{2}, m_F = \pm\frac{9}{2}) \rightarrow$ $^3P_0(F = \frac{9}{2}, m_F = \pm\frac{9}{2})$ get alternately measured



Excerpt from the level scheme for a strontium optical lattice clock[5]

$\Rightarrow$ Transition frequency

$$f_0 = \frac{f_+ + f_-}{2}$$

$\Rightarrow$ Cancelation of the first order Zeeman shift by realizing virtual spin-zero atoms