Hauptseminar: Physics of cold gases
Alkaline-earth SU(N)

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(Dated: 16.07.2013)

AN OPTICAL LATTICE CLOCK

The quality factor or Q-factor

$$Q = \frac{\nu_0}{\Delta\nu}$$

characterizes an oscillator, while the fractional instability is given by the Allan deviation

$$\sigma \approx \frac{1}{Q\sqrt{N_{\mathrm{at}}\tau}}.$$

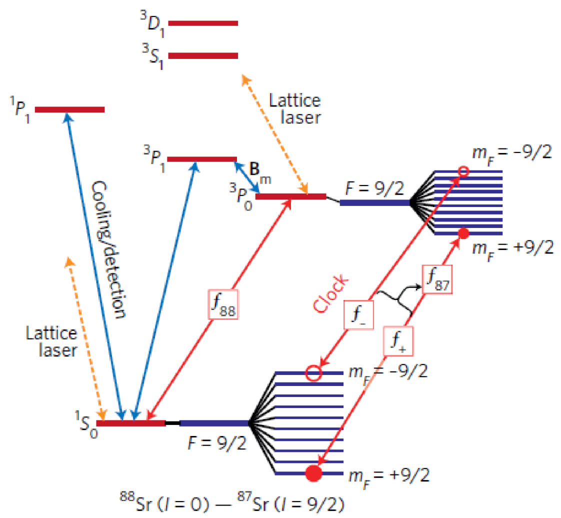


Figure 1. Level scheme for a bosonic as well as a fermionic strontium lattice clock [1]

The transition from $^1S_0$ to $^3P_0$ is used as the clock transition, as it is doubly forbidden and therefore ultra-narrow:

$\Rightarrow$ Transition from the spin- singlet to the spin-triplet

$\Rightarrow J = 0 \rightarrow J' = 0$

Admixture of the $^3P_1$-state due to

- hyperfine interactions for fermionic isotopes
- an external magnetic field for bosonic isotopes

enables the clock transition.

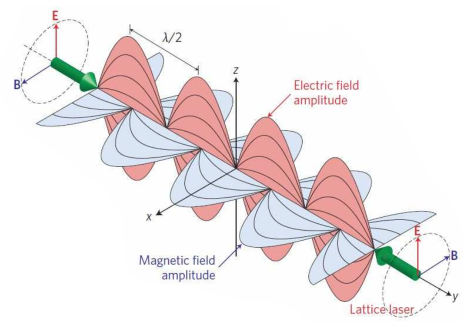


Figure 2. 1D optical lattice created by superimposing two laser beams [1]

The dipole force points to the

- intensity maxima for red detuned light
- intensity minima for blue detuned light

The clock frequency ν gets modified due to the ac-Stark effect

$$\nu = \nu_0 - [\alpha_{\mathrm{e}}(\omega_{\mathrm{L}}) - \alpha_{\mathrm{g}}(\omega_{\mathrm{L}})]\frac{E_{\mathrm{L}}^2}{4h} + O(E_{\mathrm{L}}^4)$$

with the polarizability

$$\alpha_a(\omega_{\mathrm{L}}) = \sum_b \frac{|\langle a|\underline{D}\cdot\underline{\epsilon}|b\rangle|^2}{E_b - E_a - \omega_{\mathrm{L}}} + \sum_b \frac{|\langle a|\underline{D}\cdot\underline{\epsilon}|b\rangle|^2}{E_b - E_a + \omega_{\mathrm{L}}}.$$

At certain wavelengths, the so called magic wavelengths, is $\alpha_{\mathrm{e}}(\omega_{\mathrm{L}}) = \alpha_{\mathrm{g}}(\omega_{\mathrm{L}})$ and the ac-Stark shift cancels out.

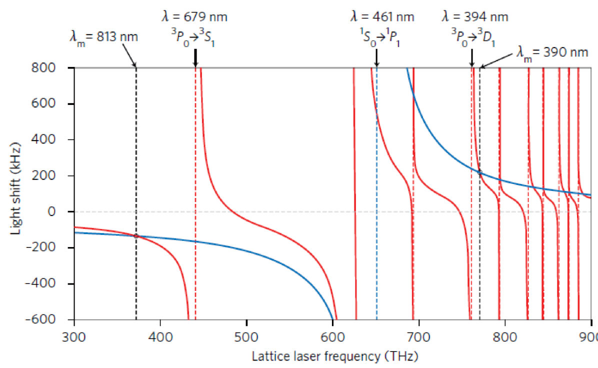


Figure 3. Light shifts of various transitions in dependence of the lattice laser frequency for the determination of the magic wavelengths [1]

COLD ALKALINE-EARTH FERMIONS IN OPTICAL LATTICES

Fermionic alkaline-earth atoms trapped in the lowest band of an optical 2D-lattice are described by

$$\begin{aligned} H =& \sum_{\alpha m} \int \mathrm{d}^3\underline{r}\,\psi^\dagger_{\alpha m}(\underline{r})[-\frac{\hbar^2}{2M}\nabla^2 + V_\alpha(\underline{r})]\psi_{\alpha m}(\underline{r}) \\ &+ \hbar\omega_0 \int \mathrm{d}^3\underline{r}[\rho_e(\underline{r}) - \rho_g(\underline{r})] + \frac{g^+_{eg} + g^-_{eg}}{2} \int \mathrm{d}^3\underline{r}\rho_e(\underline{r})\rho_g(\underline{r}) \\ &+ \sum_{\alpha,m<m'} g_{\alpha\alpha} \int \mathrm{d}^3\underline{r}\rho_{\alpha m}(\underline{r})\rho_{\alpha m'}(\underline{r}) \\ &+ \frac{g^+_{eg} - g^-_{eg}}{2} \sum_{mm'} \int \mathrm{d}^3\underline{r}\psi^\dagger_{gm}(\underline{r})\psi^\dagger_{em'}(\underline{r})\psi_{gm'}(\underline{r})\psi_{em}(\underline{r}) \end{aligned}$$

with $\alpha \in \{|g\rangle, |e\rangle\}$ and $m \in \{-I, ..., I\}$.

The field operator is defined as

$$\psi_{\alpha m} = \sum_j \omega_\alpha(\underline{r} - \underline{r}_j)c_{j\alpha m},$$

where $\omega_\alpha(\underline{r} - \underline{r}_j)$ are the Wannier functions.

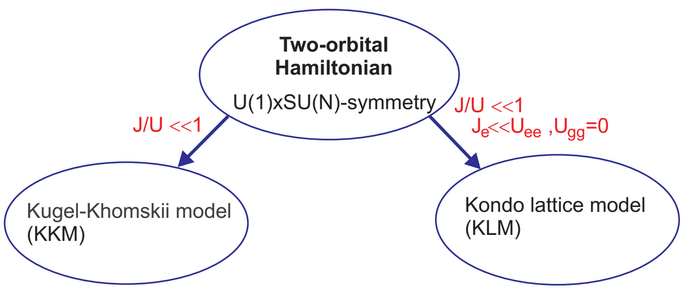
This results in the two-orbital single-band Hubbard Hamiltonian

$$\begin{aligned} H =& -\sum_{\langle j,i\rangle\alpha,m} J_\alpha(c^\dagger_{i\alpha m}c_{j\alpha m} + h.c.) + \sum_{j,\alpha}\frac{U_{\alpha\alpha}}{2}n_{j\alpha}(n_{j\alpha} - 1) \\ &+ V\sum_j n_{je}n_{jg} + V_{ex}\sum_{j,m,m'} c^\dagger_{jgm}c^\dagger_{jem'}c_{jgm'}c_{jem} \end{aligned}$$

with the energies

- J_α: Tunnelling between two neighbouring lattice sites
- $U_{\alpha\alpha}$: Onsite interactions
- V: Direct interactions
- V_{ex}: Exchange interactions

LIMITS OF THE HUBBARD HAMILTONIAN



Kugel-Khomskii model:

- Magnetic properties of transition-metal compounds are described based on superexchange
- Superexchange is the strong coupling between two magnetic non-neighbouring ions through a non-magnetic ion

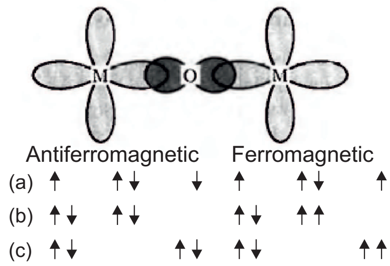


Figure 4. Transition metal oxide with possible spin configurations, (b) and (c) are forbidden in the ferromagnetic case due to the Pauli exclusion principle[2]

Kondo lattice model:

$$H = -\sum_{\langle i,j\rangle m} J_g(c^\dagger_{igm}c_{jgm} + h.c.) + V_{ex}\sum_{j,m,m'} c^\dagger_{jgm}c^\dagger_{jem'}c_{jgm'}c_{jem}$$

The Kondo lattice model describes the spin-spin interactions between the free conduction electrons and the magnetic impurities at site j.

⇒This model explains the Kondo effect:

- Resistance minimum at a non-zero temperature
- spin-antisymmetric states between conduction electrons and impurties for $V_{ex} < 0$

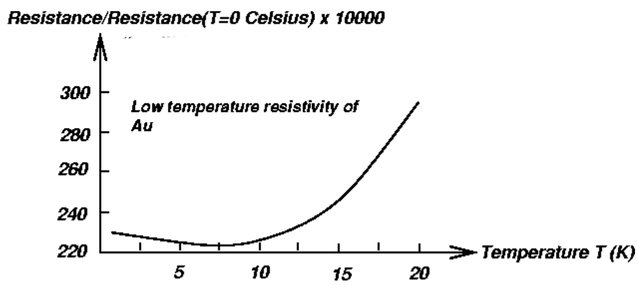


Figure 5. Temperature dependent resistance of gold with impurities[3]

[1] Hidetoshi Katori, Nature Photonics 5, 203–210(31. March 2011)
[2] Stephen Blundell, *Magnetism in condensed matter*, Oxforxd University Press 2001
[3] Wikipedia (http://en.wikipedia.org/wiki/File:Classickondo.png)