Artificial Gauge Fields for Neutral Atoms

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Outline

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What are artificial gauge fields?

- Neutral cold atoms as toy model for various physical problems
  $\rightarrow$ e.g. simulating electrons in a solid
- Condensed matter physics: easy application of magnetic fields
- Not all regimes are accessible

What are artificial gauge fields?

- Cold atoms can be controled with different methods
- Available tools: lasers, magnetic and electric fields, optical lattices,...
- Problem: atoms are neutral
  $\rightarrow$ Real magnetic fields don't influence them
- How to simulate effects on charged particles ?
- Artificial gauge fields: affect neutral atoms like a real magnetic field affects charged particles !

Example: Quantum Hall Effect

- 2D-System of electrons, e.g. AlGaAs-GaAs heterojunction
- Strong perpendicular magnetic field → resistivity ρ_{xy} is quantised
- $\rho_{xy} = \frac{1}{n}\frac{h}{e^2}$, n = 1,2,3,4,...

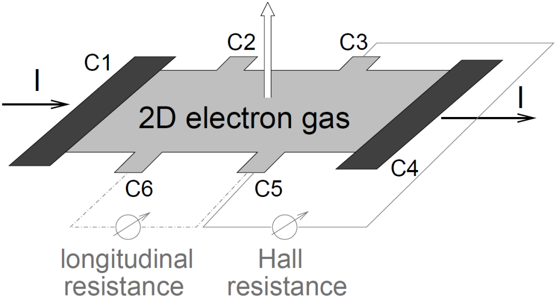
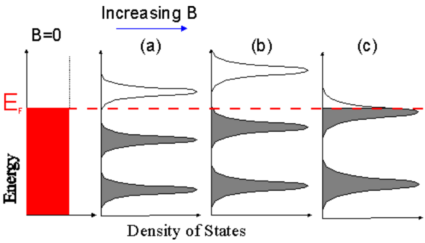


Figure : [4]

Example: Quantum Hall Effect



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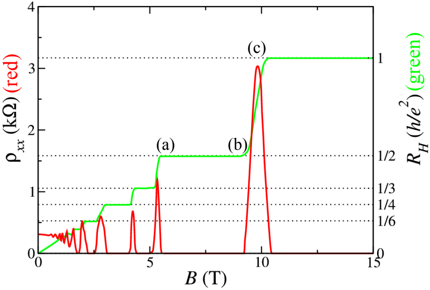


Figure : [1]

- Integer Quantum Hall Effect: explanation with Landau levels
- Fractional Quantum Hall Effect: more complicated
  $\rightarrow$ Investigate this regime more closely with cold atoms

Outline

From Rotation to Artificial Gauge Potential

- Charged particle in electromagnetic potentials:

$$H = \frac{\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2}{2m} + qV(\vec{r},t) \tag{1}$$

- Rotating neutral atoms in harmonic trap:

$$H = \frac{\vec{p}^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \vec{\Omega}\vec{L} \tag{2}$$

- (2) can be transformed to (1)
- What are q, $\vec{A}(\vec{r},t)$, and $V(\vec{r},t)$?

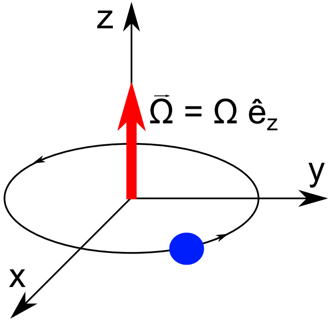
Transformed Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + \frac{1}{2} m \omega^2 r^2 - \vec{\Omega}\vec{L} = \frac{\left(\vec{p} - \vec{A}(\vec{r})\right)^2}{2m} + \frac{1}{2} m \left(\omega^2 - \Omega^2\right) r^2 \quad (3)$$

- Resulting parameters:
  $q = 1$

  $V(\vec{r}) = \frac{1}{2} m \left(\omega^2 - \Omega^2\right) r^2$

  $\vec{A}(\vec{r}) = \Omega m \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$



- Artificial magnetic field: $\vec{B} = \nabla \times \vec{A} = 2\Omega m \hat{e}_z$

Outline

Overview and principle

- BEC of ^{87}Rb, $F = 1$
- Use lasers and magnetic field to change eigenstates
- Use dispersion relation for k_x as effective Hamiltonian:

$$E(k_x) \approx \frac{\hbar^2 (k_x - k_{min})^2}{2m^*}$$

- Artificial gauge potential $A_x^* = \hbar k_{min}$

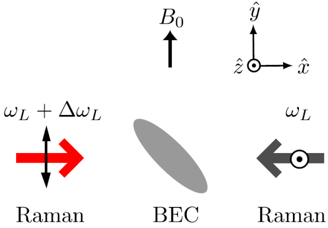


Figure : [8]

Zeeman splitting and Raman coupling

- Constant magnetic field B_0 along y-axis
  $\rightarrow$ splitting of m_F states
- $\omega_Z = g\,\mu_B\,B_0/\hbar$
- Raman lasers along x-axis
  $\rightarrow$ transitions between m_F states
- Raman detuning $\delta = \Delta\omega_L - \omega_Z$
- $\Delta\hbar k_x = \pm 2\,\hbar k_L$
- Coupling of:
  $|m_F = 1, k_x - 2\,k_L\rangle$
  $|m_F = 0, k_x\rangle$
  $|m_F = -1, k_x + 2\,k_L\rangle$

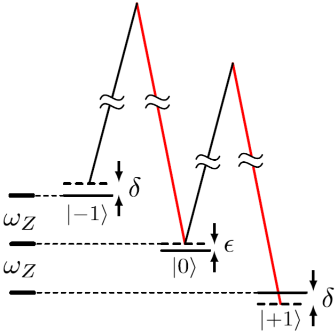


Figure : [8]

$\rightarrow$ New eigenstates with interesting dispersion relations

Hamiltonian

- Full Hamiltonian:

$$H = H_1(k_x) + \left[\frac{\hbar^2 \left(k_y^2 + k_z^2\right)}{2m} + V(\vec{r}) \right] \otimes \mathbb{1} \qquad (4)$$

with

$$H_1(k_x) = \hbar \begin{pmatrix} \frac{\hbar}{2m}(k_x + 2k_L)^2 - \delta & \Omega_R/2 & 0 \\ \Omega_R/2 & \frac{\hbar}{2m}k_x^2 - \varepsilon & \Omega_R/2 \\ 0 & \Omega_R/2 & \frac{\hbar}{2m}(k_x - 2k_L)^2 + \delta \end{pmatrix}$$

Wavevector of Raman lasers	$k_L = \frac{2\pi}{\lambda}$
Raman detuning	$\delta = \Delta\omega_L - \omega_Z$
Quadratic Zeeman shift for $m_F = 0$	ε
Raman Rabi frequency	Ω_R

Rabi frequency and dispersion relations

- Determine Ω_R for given δ
  - Black: $m_F = -1$
  - Red: $m_F = 0$
  - Blue: $m_F = -1$
- Recoil energy $E_r = \frac{\hbar^2 k_L^2}{2m}$

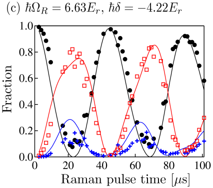
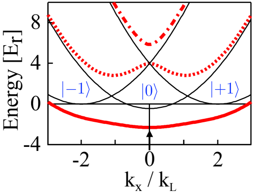


Figure : [8]

- Dispersion relations for $\delta = 0$ (left) and $\delta < 0$ (right)



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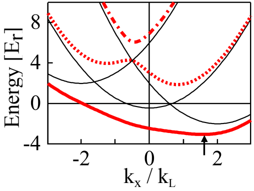

Figure : [8]

Synthetic vector potential

- Expand dispersion relation around minimum:

$$E(k_x) \approx \frac{\hbar^2 (k_x - k_{min})^2}{2m^*} \quad (5)$$

- $\hbar k_{min}$ resembles x-component of a vector potential
- Problem: $k_{min} = \text{const.}$ for $\delta = \text{const.}$
  $\rightarrow$ Resulting synthetic magnetic field $\vec{B} = \nabla \times \vec{A} = 0$

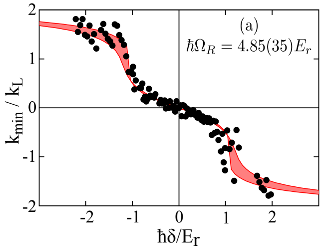

Figure : [8]

Creating a non-zero synthetic magnetic field

- New setup geometry $\rightarrow k_L = \frac{2\pi}{\lambda\sqrt{2}}$
- Varying real magnetic field $\vec{B}(y) = (B_0 - b'y)\,\hat{e}_y$
- $\delta = \Delta\omega_L - \omega_Z$

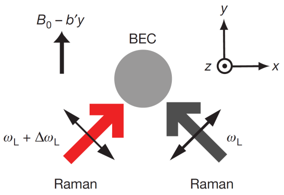

Figure : [7]

- $\omega_Z = g\,\mu_B\,B(y)/\hbar$
- Detuning gradient $\delta' = \frac{\partial\delta}{\partial y} = g\,\mu_B b'/\hbar$
- Synthetic vector potential $A_x^* = A_x^*(\delta)$

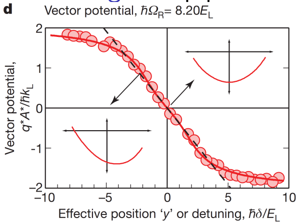

Figure : [7]

$\rightarrow$ Synthetic magnetic field $B^* = -\frac{\partial A_x^*}{\partial y} = -\delta' \frac{\partial A_x^*}{\partial \delta}$

Experimental proof: formation of vortices

- Absorption-imaging after time-of-flight

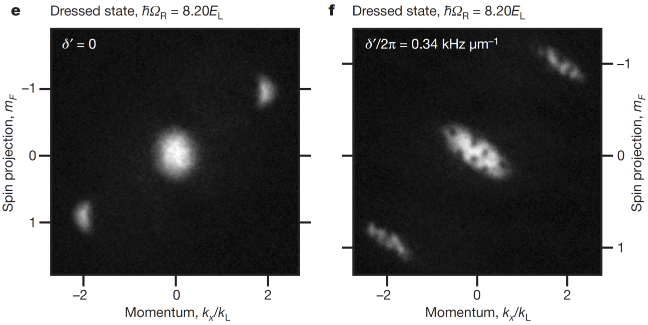

Figure : [7]

- Stern-Gerlach effect: spin components are separated along y

Experimental proof: formation of vortices

- Increasing detuning gradient δ' $\rightarrow$ more vortices

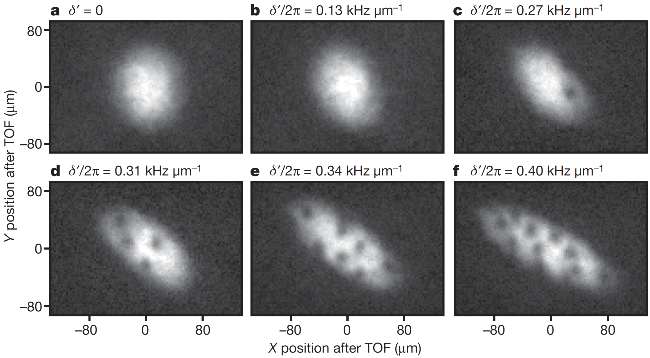

Figure : [7]

Outline

Bloch electrons in strong magnetic fields

- Electrons in a periodic potential
  $\rightarrow$ Bloch waves: $\psi_{n\vec{k}}(\vec{r}) = u_{n\vec{k}}(\vec{r})\, e^{i\vec{k}\vec{r}}$
- 2D-square-lattice (x-y-plane) with lattice spacing a
- Bloch energy function:
  $W\left(\vec{k}\right) = 2E_0\,(\cos(k_x a) + \cos(k_y a))$

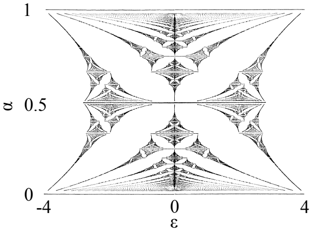

Figure : [5]

- Harper's equation:
  $g(m+1) + g(m-1) + 2\cos(2\pi m\alpha - \nu)\, g(m) = \varepsilon\, g(m)$
- Douglas R. Hofstadter: Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields, Phys. Rev. B, 14, Published 3 September 1976

Hofstadter butterfly and sceleton

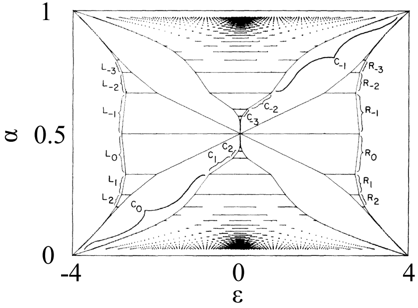

Figure : [5]

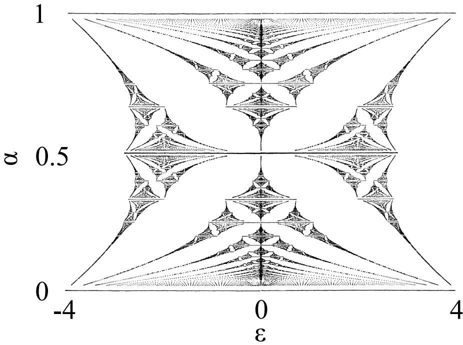

Figure : [5]

- $\alpha = \frac{a^2 B}{2\pi(\frac{\hbar c}{e})} = \frac{1}{2\pi}\frac{\Phi_B}{\Phi_0} = \frac{1}{2\pi}\Delta\varphi \qquad \varepsilon = \frac{E}{E_0}$
- $\Delta\varphi =$ phase gained when moving around one lattice cell

Proposal for experimental realisation

- Cold atoms in 2D optical lattice
- Description with Bose-Hubbard-model
- Trap different internal states in different columns of lattice
- Induce hopping amplitudes of same magnitude along x- and y-axis
- Non-vanishing phase for atoms moving around a lattice cell

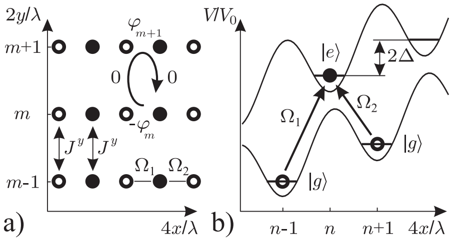

Figure : [6]

Proposal for experimental realisation

- Kinetic energy induced hopping only along y-axis
- Add linear potential in x-direction
  - Accelerating the lattice: $\mathrm{H_{acc}} = \mathrm{M}\, a_{\mathrm{acc}} \mathrm{x}$
  - Electric field $\mathrm{E} = \mathrm{E}'\mathrm{x}$: $\mathrm{H_{acc}} = \mu\, \mathrm{E}'\mathrm{x}$
- Generate spacially varying Rabi frequencies $\Omega_{1,2}$

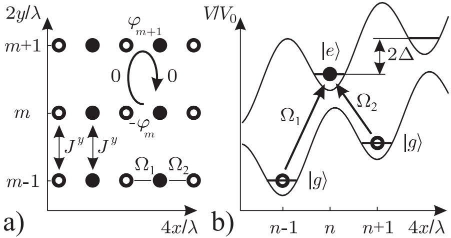

Figure : [6]

Proposal for experimental realisation

- Lasers create y-dependent phase $\varphi_m = q\, m\, \frac{\lambda}{2}$
- q: y-component of wavevector (of coupling lasers)
- Phase gained when moving around a lattice cell:
  $\Delta\varphi = \varphi_{m+1} - \varphi_m = q\, \frac{\lambda}{2} = 2\pi\alpha$
- $\alpha = \frac{q\lambda}{4\pi}$

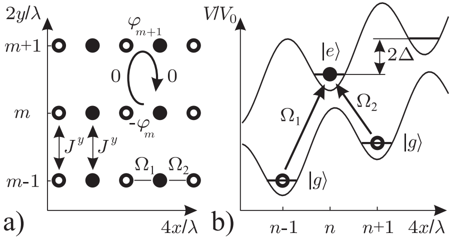

Figure : [6]

Proposal for experimental realisation

- Small filling factors $\bar{n} << 1 \rightarrow$ neglect interaction terms
- Hopping amplitudes $J^x \approx J^y$
- Hamiltonian:
  $H(\alpha) = J\sum_{m,n}\left(e^{2\pi i \alpha m} c^\dagger_{n,m} c_{n+1,m} + c^\dagger_{n,m} c_{n,m+1} + h.c.\right)$
- Equivalent to H for electron moving on lattice with magnetic field $B = \frac{2\pi\alpha}{A\ e}$ with $A = a_x a_y$ (area of one lattice cell)

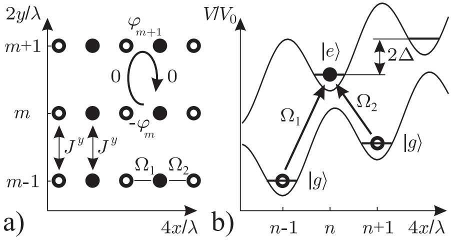

Figure : [6]

Outline

Summary

- Why do we need artificial magnetic fields for neutral atoms ?
  $\rightarrow$ Quantum-Hall effect
- Engineering Hamiltonians for neutral atoms
  $\rightarrow$ artificial vector potential
  - Rotating BEC
  - Light-induced vector potential
- Theory of the Hofstadter butterfly
- Possible experimental realisation

Thank you for your attention !

Advanced Quantum Mechanics II.
http://oer.physics.manchester.ac.uk/

Quantum Hall Effect.
http://homepages.warwick.ac.uk/

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